C. O. HARRIS

Slide Rule

SIMPLIFIED

SLIDE RULE SIMPLIFIED

By Charles O. Harris

Many people have been frightened away from using this valuable tool by the idea that it is difficult to master. This book recognizes the usual difficulties of the beginner and overcomes them. The treatment is so thorough and simplified that anyone who can multiply and divide can easily master the slide rule.

Comments from the Press

"This 266-page book contains many pictorial views of the slide rule actually in use and the various operations and points under discussion are clearly emphasized. The book will prove helpful to people who want to use the slide rule only for common calculations, as well as for those who want to use it for advanced mathematics. People who have had no mathematics beyond the eighth grade will be able to learn from this practical book. It shows them how to multiply and divide, as well as find the square and square root, cube and cube root, etc. In fact, the first eight chapters are devoted to such common calculations. More advanced work is covered in succeeding chapters."

From:
Hitchcock's Machine Tool Blue Book

"The book should be of great value to both beginners and those having previous slide rule experience, for the author has given excellent instruction in how to use a slide rule to its greatest advantage. The first eight chapters discuss fundamental arithmetic operations—multiplication, division, the square, square root, the cube and cube root. Foreseeing the usual difficulties of the beginner, the author has emphasized accurate and precise reading of the scales and locating the decimal point. Careful study of the first eight chapters of this book should enable any person with a knowledge of elementary arithmetic calculations to perform them on the slide rule. The remainder of the book takes up more advanced operations and should be of interest to those having previous experience with a slide rule or those intending to use higher mathematics. Each operation is clearly explained and illustrated by practical examples. Sketches picturing the steps in examples make the explanations even more complete. Since practice is essential in acquiring knowledge and skill in using a slide rule, many practice problems have been furnished. The problems are representative of the type which the reader might solve with a slide rule. Rules are given for locating the decimal point by means of the 'digit count' system. Chapters are followed by review problems which are useful as class examinations or self-tests. For those who enjoy mathematics or who wish to know why the slide rule works as it does, the basis of each operation is explained by means of logarithms. The text was written specifically for the use of a 10-in. Mannheim rule, for this is the simplest and most widely used rule, and illustrations in the book were prepared from this rule. However, the operation of all slide rules is essentially the same. Variations found on other types of slide rules are discussed following the description of the operation in which they are used."

From:
American Foundryman

Book No. 906

SLIDE RULE SIMPLIFIED

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AMERICAN TECHNICAL SOCIETY

CHICAGO, U.S.A.

1st Printing 1943
2d Printing 1944
3d Printing 1945
4th Printing 1946
5th Printing 1949
6th Printing 1951
7th Printing 1952
8th Printing February, 1953
9th Printing October, 1953
10th Printing 1954
11th Printing 1955

Printed in the U.S.A.

PREFACE

The slide rule is a valuable aid to the estimator, merchant, accountant, manufacturer, and to other business men, and is an indispensable tool to the engineer and scientist. To all these it is conspicuous as a timesaver. Many people who would like to use it are frightened away by the idea that operation of the slide rule is difficult to master. One of the primary aims of this book is to dispel this idea, and replace it with the certain knowledge that anyone who will study and practice can learn to use the slide rule with ease and confidence.

The instructions are presented in a manner that recognizes the usual difficulties of the learner and overcomes them. An unusual feature of the book is the complete instruction on how to read the different scales of the slide rule accurately and precisely and thus forestall serious errors. Each type of calculation, multiplication, division, combinations of multiplication and division, the square and square root, the cube and cube root—all are discussed so thoroughly that they can be mastered by the reader who studies alone, as well as by those who study in the classroom. The treatment of each operation in the first eight chapters is so complete that it can be followed by anyone who has studied arithmetic and can multiply two numbers and divide one number by another, even though he has never before seen a slide rule.

On the other hand, the material in the final chapters will be of interest to students who have had previous experience with the slide rule. These deal with sines and cosines, the tangent of an angle, and logs and antilogs. Only a small percentage of those who use a slide rule are well enough acquainted with it to take full advantage of its possibilities, for it has many uses not commonly known. One object of this book is to present such a thorough treatment of

the slide rule that one who studies all of it will be able to perform difficult numerical calculations easily and accurately, and with confidence.

Method of Explanation. The method of explanation is, in general, uniform throughout the book:

1. The operation, for instance multiplication, is carefully described, step by step, logically.

2. Each process is illustrated by a number of Illustrative Examples. Each of these was carefully chosen so that the explanations are unusually complete. In many cases, sketches provide a picture of each step taken in the solution of a given example.

3. A great many Practice Problems are furnished for the student. It is especially important that these problems be solved, since it is only by practice that the correct use of the slide rule is mastered and skill acquired. An important feature of these Practice Problems is the fact that they follow the form used in practical work; this means that the type of problems studied is exactly the same type that the reader might have to solve with the slide rule. Answers to these problems are given in the back of the book so that solutions may be checked.

4. In all cases exact rules are given for locating the decimal point by means of a new system of "digit counts."

5. At the end of most chapters, Review Problems are given. There are no answers. Instructors will find these Review Problems useful as Examinations for classes; students working alone should use them as self tests.

6. The basis of each fundamental operation is explained by means of logarithms, since the slide rule is based on logarithms. Study of these explanations on the basis of the process is optional. They are given for the benefit of those who enjoy mathematics for its own sake, or who want to know just why the slide rule manipulations give the correct results.

Attention is called to a brief discussion of Negative Numbers and the Law of Signs which has been put in the back of the book as a review for those who have forgotten.

A 10-inch Mannheim slide rule was selected as a basis for the text because it is the most widely-used type of slide rule, it is the

simplest and easiest to master, and any arithmetical calculation except addition and subtraction can be performed with it. The book has been written expressly for the Mannheim slide rule with CI and K scales, and the illustrations have been prepared from it; thus such a slide rule on which to practice fits the text exactly.

Slide rules of this type are made by many different manufacturers. They are essentially the same, in that the same work can be done with them; however, there are slight differences in detail. For this reason, each chapter which describes an operation with the slide rule contains, at the end, a section on other types of slide rules.

The author wishes to express his thanks and appreciation to Mr. Arthur E. Burke, Art Director of the American Technical Society, and to Mr. John Corrigan for their original and excellent work in preparing the drawings and illustrations which appear throughout the book.

CHARLES O. HARRIS

HOW TO USE THIS BOOK

It is suggested that the book be studied in the following manner:

1. Study the chapters in the order given, at least through Chapter VIII.

2. Learn each point thoroughly before going on to the next.

3. Read the explanations given and follow the Illustrative Examples carefully. Many of these examples have been suggested by common mistakes of other learners, and careful study will help you to avoid these, or similar, mistakes.

4. Work all the problems, both Practice and Review. A great deal of practice is necessary in order to become proficient in the use of the slide rule.

5. If interested in the basis of the slide rule, read the sections headed "Basis of the Process." However, it is not necessary to know the basis in order to use the slide rule. In any case, do not read the basis of an operation until you have learned the operation.

6. For convenient, ready reference, the fundamental rules have been assembled and put in the back of the book. You will also find there a review of negative numbers and the law of signs.

Practice. Practice as much as you can. As soon as you learn its operation, use the slide rule in your work whenever possible.

Decimal Point. A great deal of attention has been given to rules for locating the decimal point in the answer. This is an especially important consideration since an answer ten or one hundred times too large, or ten or one hundred times too small, is not only worthless, but can cause much harm. A system of "digit counts" is explained, which makes it possible to state exact and concise rules

for locating the decimal point, and give them in terms of the digit counts for the numbers, rather than in terms of the characteristics of the logarithms. This is simple and more direct, and enables the learner to use the slide rule without any knowledge of logarithms.

Anyone who wants to use the slide rule for unusual or advanced calculations will find the basis for his work in this textbook.

Continued practice will bring skill in setting and in reading the slide rule and will give one confidence in the results obtained.

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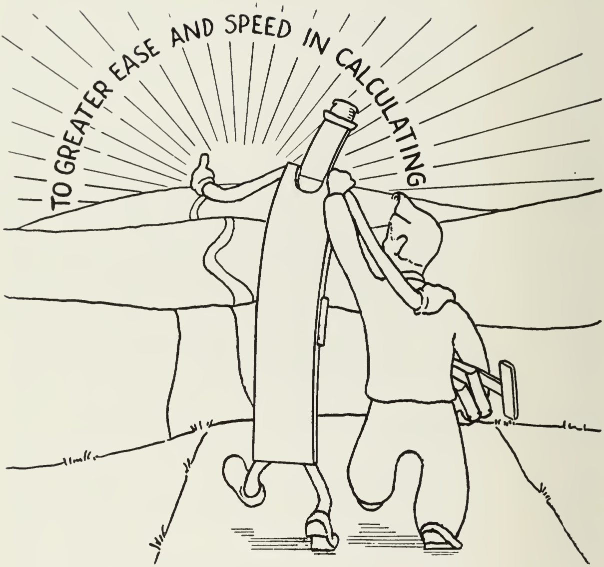


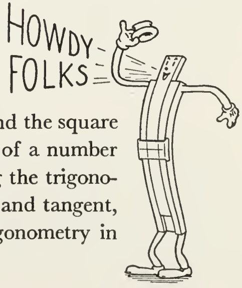
THE SLIDE RULE WILL HELP YOU DO IT

CHAPTER I

INTRODUCTION

WHAT THE SLIDE RULE IS. The slide rule is an instrument for performing certain arithmetical calculations. It is used widely by engineers, architects, businessmen, shopmen, and others who wish to make these arithmetical calculations quickly and accurately. The most important feature of its use is the saving of time and energy that it makes possible. The slide rule consists of three parts: the fixed part, called *the stock; the slide,* or movable part in the center; and *the runner.* The runner is a piece of glass or other transparent material with a scratch or hairline in the center. Fig. 1 is an illustration of a slide rule with the different parts labeled. The slide rule is similar to many other instruments, in that, by its use, work can be done more quickly and accurately than by hand. However, unlike many instruments, its use can be learned easily. Anyone who will devote a reasonable amount of time to practice can learn to use the slide rule with ease and confidence.

WHAT YOU CAN DO WITH THE SLIDE RULE. Any calculation in arithmetic except addition and subtraction can be performed with the slide rule. The most common operations are multiplication, division, and combinations of these; to find the square of a number or its square root; the cube of a number or its cube root; and operations involving the trigonometric functions such as the sine, cosine, and tangent, although it is not necessary to know trigonometry in order to use the slide rule.



ADVANTAGES OF THE SLIDE RULE. The greatest advantage of the use of a slide rule is the saving of time and energy. In most problems, the labor of making the calculations in longhand requires much more time than the determination of just what calculations to make. When the slide rule is used, it is often possible to save 90 per cent of the time otherwise devoted to calculation. For example, to find the result of

$$\frac{436 \times 76 \times 362}{372 \times 52 \times 172}$$

longhand would require making two multiplications to find the numerator, then two more to find the denominator, and finally,

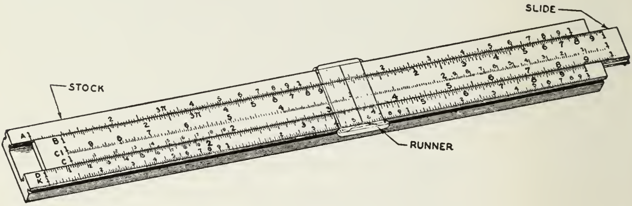


Fig. 1

dividing the numerator by the denominator. It would be necessary to write down many numbers in the intermediate steps of the process. With the slide rule, each of the six numbers would be located on the rule at the proper time in the operation, and the only number to write, besides setting down the original numbers,

$$\frac{436 \times 76 \times 362}{372 \times 52 \times 172}$$

would be the answer. The calculation could be made in 15 seconds, which represents a great saving of time. The energy consideration is equally important. Work done longhand results in the expenditure of much more mental and physical effort than the same amount of work done with the slide rule. This leads to fatigue and nervous

strain, which in turn lead to errors; all this can be largely eliminated by using the slide rule. An important feature of industrial development is the requirement that workers of each generation accomplish more in a day than those of the previous generation. This places a premium on speed and accuracy. Hence, laborsaving devices become more and more important. Time and concentration, that can be saved for any part of the task that really demands them, are too valuable to waste.

Compilation of tables usually leads to a certain type of calculation (for example, the division of the product of two numbers by the product of two others) which must be performed separately for each of many combinations of numbers. For doing such work, the slide rule offers methods which eliminate a great deal of unnecessary labor.

With the slide rule it is as easy to work with one number as another. The multiplication of 268 by 341 is as easy as 300 by 200. Multiplication with the slide rule involves only writing the operation, 268×341, a quick manipulation of the rule and writing the answer. In making this particular multiplication longhand, ten extra numerals must be written down during the process, and at least eleven mental steps are necessary. If made with the slide rule, calculations involving large numbers are no more difficult than those involving small ones. It is as easy to divide 23,700 by 198 as to divide 39 by 7.

When calculations require the use of the sine, cosine, or tangent of an angle, the function can be obtained from the slide rule. This releases the operator from the inconvenience of consulting a book of tables when in the middle of a problem, and also saves time. In an emergency when a set of tables is not available, the advantage of being able to determine the sine of an angle with the slide rule becomes most apparent.

Further advantages of the slide rule are compactness and cheapness. An eight- or ten-inch slide rule can be carried in a pocket, notebook, or brief case. Other devices for rapid calculation are either bulkier or less flexible in operation. Other mechanical calculators which compare in efficiency are far more expensive.

SUGGESTIONS FOR LEARNING. It is easy to learn to use the slide rule. A knowledge of logarithms will be found helpful but is not essential. The fundamental operations can be learned through study, and with practice soon become automatic.

The material in this book should be read with the slide rule in hand. The Illustrative Examples should be studied in detail and the problems worked carefully. There are enough problems so that anyone who works them all will become skilled in the use of the slide rule.

Speed in manipulating the slide rule will soon be acquired but should not be forced. The time saved by substituting the slide-rule method for the longhand method is far more important than the time saved by hasty operation of the rule itself.

A feeling of confidence in the results obtained by slide-rule calculation will develop rapidly. The beginner may be tempted to check his work by performing the operation longhand. With experience will come the realization that the use of the slide rule actually reduces the number of errors.

WHAT SLIDE RULE TO USE. The slide rule described in this book is satisfactory for learning purposes, and for many practical uses. It is simple and its operation is easy to learn; yet it is sufficient for all types of calculations. It is the 10-inch length, which is the most common and the most practical.

As you acquire skill and progress in understanding of the slide rule, you may feel that your knowledge and requirements justify the purchase of a better slide rule. Advantages of the better rules include:

1. Better construction, which makes manipulation easier
2. Freedom from shrinking and warping, so that the slide rule remains accurate
3. Clearer marking, which prevents eyestrain

Slide rules are available in a wide range of prices; but any slide rule, no matter what its cost, is a good investment.

In purchasing a new slide rule it is well to consider the conditions under which it will be used. It is obviously unwise to subject an expensive slide rule to dirt or mechanical hazards.

A slide rule of 8- or 10-inch length is the most convenient for

general use. The scales are long enough for precision and the rule is not too long to be carried handily in a brief case or notebook. The 20-inch slide rule gives greater precision but is not so easy to carry around and is awkward to use in the classroom.

The discussions in this book apply particularly to the 10-inch Mannheim slide rule with CI and K scales. However, the material is applicable to a slide rule of any size.

CHAPTER II

HOW TO HOLD THE SLIDE RULE

If the slide rule is to be used efficiently, it is necessary for the operator to be able to adjust the slide and runner to the proper numbers with speed and precision. This requires that the slide move easily in the stock. The slide should fit just tightly enough so that its own weight will not cause it to drop from the stock when the rule is held vertically. If the slide sticks, it can usually be made to move easily by lubricating with graphite from a soft lead pencil. Fig. 2 shows how the pencil is to be applied to the slide. In addition to providing lubrication, this process removes dirt from the corners of the slide.

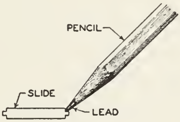


Fig. 2

ADJUSTING THE RUNNER. Most slide rule operations commence with the location of a number by means of the hairline of the runner. Fig. 3 shows the proper position of the hands on the rule for this maneuver. The rule should be held lightly and the palms of the hands should not be in contact with it. Of the front of each hand, only the cushion at the base of the little finger should touch the slide rule. This affords sufficient purchase for holding the rule and leaves the thumbs free to adjust the runner. A slight pressure on the runner and stock by each thumb will hold the runner in full control and allow it to be adjusted quickly. Most beginners hold the rule too tightly. Any tendency toward this must be overcome.

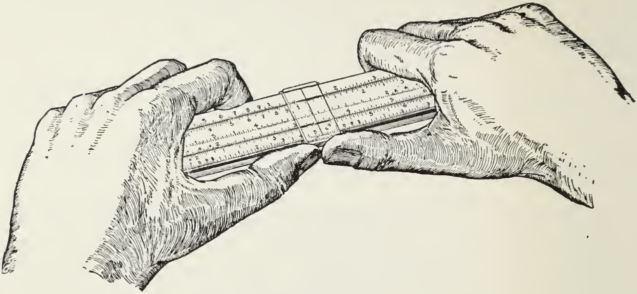


Fig. 3

When the number with which the calculation commences is near the end of the slide rule, the same spacing of the hands should be maintained as they move along the rule. As one hand moves off the rule, its forefinger may be used to assist the thumb in controlling the runner. A little practice in sliding the runner back and forth will help you to understand and follow these directions.

ADJUSTING THE SLIDE. The user of the slide rule must be able to adjust the slide to its proper position for each calculation. Fig. 4 shows the proper position of the hands on the rule when the right end of the slide projects from the stock. The left hand holds the stock lightly and the forefinger pushes against the slide. The right hand grips the slide with the forefinger pressing against the stock. The palms of the hands should not touch the

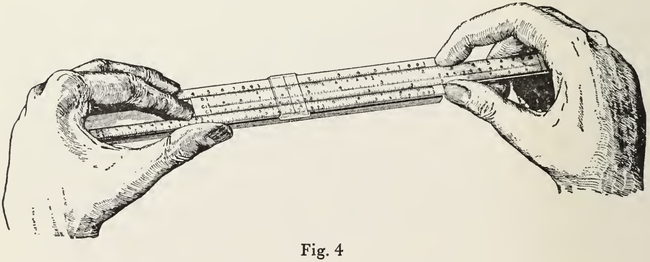
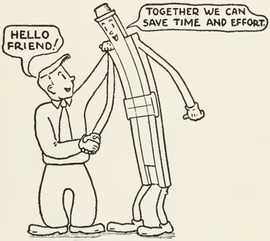


Fig. 4

slide rule. When the slide rule is held in this manner, the slide can be adjusted quickly and precisely.

When the left end of the slide projects from the stock, the positions of the hands are interchanged.

The scales on the back of the slide are read by turning the rule over and extending the slide to the right. For this, the left hand should be held as in Fig. 3 and the right hand as in Fig. 4.



TREAT THE SLIDE RULE AS A FRIEND

CHAPTER III

THE SCALES OF THE SLIDE RULE

All slide rule calculations, multiplication, division, etc., involve locating, in their proper positions on the rule, the numbers with which the operation is to be performed; and reading, on the slide rule, the number which is the answer. In order to multiply 16×13, for instance, you must know exactly how and where the numbers 16 and 13 are to be located on the slide rule, and where the answer will be found and how to read it. The first step in learning to use the slide rule is to become familiar with all parts of it so that locating a number or reading an answer can be done quickly and accurately. The purpose of this chapter is to describe and explain the different parts of the slide rule so that you will be able to do this. The chapter should be read with the slide rule at hand. All examples should be followed on your own rule.

SCALES. Fig. 1 shows the front of the slide rule. The fixed part, or frame, is called the *stock,* the movable part in the center is called the *slide,* and the glass or celluloid is called the *runner.* The scratch or hairline on the runner is used to locate and read numbers on the slide rule. Each of these parts is labeled. Work carefully and be sure that you understand each topic before going on to the next. Nothing can be gained by skipping around.

DON'T WRESTLE THE SLIDE RULE

The slide rule in Fig. 1 has six rows of marks, each row extending along, and parallel to, the rule. Each row is called a *scale* and bears the name of the letter at its left end. Thus the A scale, with the letter A at its left end, is on the upper part of the stock; the

B, CI (pronounced see-eye), and C scales are on the slide; and the D and K* scales are on the lower part of the stock.

Part of the back of the slide rule is shown in Fig. 5. This figure has been drawn with the slide extending to the right of the stock, so that part of the three scales on the back of the slide appear. Each of these scales consists of a row of marks parallel to the slide and each is designated by a letter at the right end. From top to bottom in Fig. 5, the scales are:

1. The sine scale, designated by the letter *S*
2. The log scale, designated by the letter *L*
3. The tangent scale, designated by the letter *T*

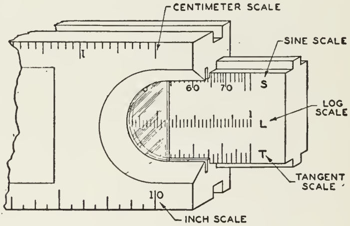


Fig. 5

You will notice a small piece of glass or celluloid set into the back of the slide rule at the right end. Some rules have a mark on the celluloid and this mark is used to locate a number on a scale. If there is no mark, the edge of the celluloid is to be used instead.

THE DIGITS OF A NUMBER. Numbers consist of various sequences of the numerals *0* to *9* inclusive. For example, the numbers *3495, 2.16,* and *0.00948* are sequences of numerals. For the purpose of the discussions which follow in this book, *the first digit of a number is defined as the first numeral (other than zero) as the number is read from left to right.* In determining the first digit, any zeros at the extreme left of the number are passed over. In each of the following numbers, the first digit appears in

*Not all slide rules have the CI and K scales. However, as so many do, a book of this type would not be complete without discussions of their use. They make it possible to save a great deal of time on certain calculations.

bold-face type: 0.000**3**78; **7**.85; **4**3200; 0.0**2**09; **5**76; **1**,000,000; 0.**1**003. The first digit cannot be zero. The second digit is the numeral immediately after the first digit and the second digit can be zero. The third, fourth, fifth digits, etc., are the numerals in order as they follow the second digit of the number. Any digit except the first can be zero. The following examples will illustrate this.

ILLUSTRATIVE EXAMPLES

1. In the number 0.9083, the first digit is *9,* the second digit is *0,* the third is *8,* and the fourth is *3.*

2. In the number 12.06, the first digit is *1,* the second digit is *2,* the third is *0,* and the fourth is *6.*

3. In the number 35000, the first digit is *3,* the second digit is *5,* and the third, fourth, and fifth digits are each zero.

4. In the number 1.933, the first digit is *1,* the second digit is *9,* and the third and fourth digits are each *3.*

5. In the number 0.00628, the first digit is *6,* the second digit is *2,* and the third digit is *8.* The zeros to the left of *6* are not counted as digits.

6. In the number 1.0004, the first digit is *1,* the second, third, and fourth digits are each *0,* and the fifth digit is *4.*

THE C AND D SCALES

The C and D scales, which are identical, appear on the lower part of the front of the slide rule. (See Fig. 1.) The C scale is on the slide, or movable part in the center, and the D scale is on the fixed part or stock.

USE OF THE C AND D SCALES. The C and D scales are used together for multiplication and division. In using the slide rule, multiplication and division are the most important operations because they are those most frequently performed. The C and D scales are also used with the A and B scales for finding the square, square root, cube, or cube root of a number; with the K scale for finding the cube or cube root of a number; with the tangent scale for finding the tangent of an angle or arctangent of a number; and with the log scale for finding the log or antilog of a

number. The C and D scales are used more than any of the other scales on the slide rule.

MARKING OF THE C AND D SCALES. The number *1* at the left end of each scale is called the *left index,* see Fig. 6. The number *1* at the right end is called the *right index.* Between the two indices are the divisions, numbered *2* (about three-tenths of the total length from the left index), *3, 4, 5, 6, 7, 8,* and *9.* Hereafter the word *division* will mean one of these marks. Each length between two consecutive divisions, for example between the divisions *4* and *5,* is divided into ten *sections* and each section is sub-

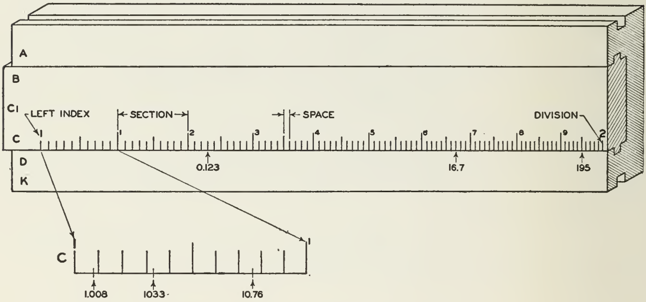


Fig. 6

divided into *spaces.* Between the left index of the scale and the division marked *2,* each section has ten spaces. This portion of the scale is shown in Fig. 6 where the terms *division, section,* and *space* are illustrated. Between the divisions marked *2* and *4,* there are only five spaces to each section, and to the right of the division marked *4,* there are only two spaces to each section.

DECIMAL POINT. The position of the decimal point of a number has nothing to do with the location of the number on the C or D scales of the slide rule. No matter where the decimal point is, the number will have the same location on the C or D scale. The only thing that influences the location of the number is the sequence of digits in the number. Thus the numbers *0.000348; 0.348; 3.48; 348; 34,800,* would all be the same on these scales.

HOW TO LOCATE A NUMBER ON THE C AND D SCALES. When the First Digit is 1. Fig. 6 shows the portion of the C scale between the left index and the division marked *2*. Any number which has *1* for its first digit is located here. The second digit of the number determines in which one of the ten sections the number is to be located, and the third digit locates the number in the proper space of the section. Any digit except the first may be zero. The first cannot be zero, but is defined as the first numeral of the number which is not zero. Correct locations for the numbers *0.123, 16.7* and *195* are shown in Fig. 6. Detailed instructions for locating these numbers follow:

ILLUSTRATIVE EXAMPLES

7. To locate the number 16.7.

1) Since the first digit is *1,* the number is located in the portion of the scale shown in Fig. 6.
2) The second digit is *6,* so count off six sections from the left index. Note that the mark at the right of the sixth section bears the number *6.*
3) From this mark, count off seven spaces, since the third digit is *7.* The number is located at the right end of the seventh space.

8. To locate the number 195.

1) The fact that the first digit of the number is *1* shows that the number is to be located between the left index and the division marked *2.*
2) Count nine sections from the left index since the second digit is *9.*
3) From here count five spaces to the right. The number is located at the end of the fifth space beyond nine sections.

9. To locate the number 0.123.

1) The first digit of the number is *1* so the number is located in the portion of the scale between the left index and the division marked *2.*
2) The second digit is 2, so count two sections from the left index.
3) From here, count three spaces to the right. This is the location of the number.

Numbers having more than three digits. Any number which has *1* for its first digit is located between the left index of the scale and the division marked *2.* If the number has no more than three digits it can be located exactly, that is, it will fall on a mark between two spaces. This was the case in each of the three foregoing examples. A number having four digits must be located approximately, since it will be between the marks. The first three digits locate the number in the proper space. The fourth digit is the number of tenths of a space between the left end of the space and the location of the number. For example, if the fourth digit is *6,* the number is to be located six-tenths of the space from the left side of the space, and this six-tenths of the space must be estimated. Fig. 6 also shows a larger view of the section at the extreme left of the scale with correct locations for the numbers *1.008, 1033* and *10.76.* The method of locating these numbers is described in the following examples:

ILLUSTRATIVE EXAMPLES

10. To locate 1.008.

1) Since the first digit is *1,* the number is located between the left index of the scale and the division marked *2.*
2) The second digit is *0,* so the number lies in the first section.
3) The third digit is also *0,* so the number lies in the first space of this section.
4) Last, estimate eight-tenths of this space, since the fourth digit is *8.* This number is located here.

11. To locate 10.76.

1) The first digit is *1,* so the number lies between the left index of the scale and the division marked *2.*
2) Since the second digit is *0,* the number lies in the first section.
3) The third digit is *7,* so count off *7* spaces from the left index.
4) Since the fourth digit is *6,* estimate six-tenths of a space beyond this and locate the number there.

12. To locate 1033.

1) The first digit is *1,* which shows that the number lies between the left index and the division marked *2.*
2) The second digit is *0,* so the number lies in the first section.

3) The third digit is *3,* so count three spaces from the left index.
4) Since the fourth digit is *3,* estimate three-tenths of a space from the left side of the space in which the number belongs.

For any number which has *1* for its first digit, only the first four digits can be considered in locating the number on the C and D scales. If there are more than four digits, those after the fourth are treated as though they were zeros. For example, the number *157836* would be set as *157800* on the slide rule. This is an approximation, but the error involved is less than one part in one thousand. There are few engineering or shop calculations in which such an error

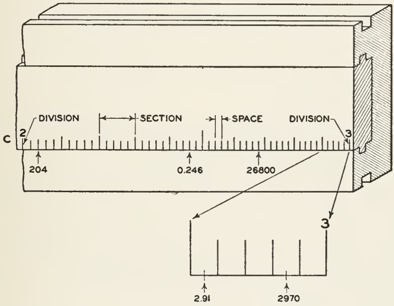


Fig. 7

would be serious. In many cases it would be necessary to increase the fourth digit when treating those after it as zeros in order to get the best approximation. For instance, the number *11389* is closer to *11390* than it is to *11380,* so *11390* would be used. When the fifth digit is *5* or greater, the fourth digit should be increased by one when replacing the digits after it by zeros; when the fifth digit is less than *5,* the fourth digit should be left unchanged.

When the First Digit Is 2. Any number having *2* for its first digit is located between the division marked *2* and the division marked *3.* This portion of the scale is shown in Fig. 7 with illustrations of a section and a space. The second digit shows in which section the number lies. Since there are only five spaces in each section, each space has a value of two in the third digit. Thus the

third digit of a number located in this portion of the scale can be set precisely only if it is even. The numbers *204, 0.246* and *26800* are shown in their proper locations in Fig. 7. The process of locating each of these numbers is shown in the following examples.

ILLUSTRATIVE EXAMPLES

13. To locate 204.

1) Since the first digit is *2,* the number is located between the divisions marked *2* and *3.*
2) The second digit is *0,* so the number is located in the first section of this portion of the scale.
3) The third digit is *4,* but since each mark on the scale has a value of two in the third digit, you count only two spaces from the left of the section to locate the number.

14. To locate 0.246.

1) The first digit of the number is *2,* so the number is located between the divisions marked *2* and *3.*
2) The second digit is *4,* so count four sections from the division marked *2.*
3) From here count three spaces since the third digit is *6* and each space has a value of two in the third digit.

15. To locate 26800.

1) The number is located between the divisions marked *2* and *3* since the first digit is *2.*
2) Since the second digit is *6,* count six sections from the division marked *2.*
3) From here count four spaces since the third digit is *8* and each space has a value of two in the third digit.

Since the fourth and fifth digits are zero, they are not used in locating the number.

If the third digit of the number is odd, it can be located by estimating one-half of a space. A larger view of the section at the extreme right of this portion of the scale is also shown in Fig. 7 with the proper locations for the numbers *2.91* and *2970.* The procedure for locating each of these numbers is given in the examples.

ILLUSTRATIVE EXAMPLES

16. To locate 2.91.

1) The first digit of the number is *2,* so the number is located between the divisions marked *2* and *3.*
2) Since the second digit is *9,* count *9* sections from the division marked *2.*
3) From here, estimate one-half of a space to the right. This locates the number, since the third digit is *1* and each space has a value of two in the third digit.

17. To locate 2970.

1) The number is located between the divisions marked *2* and *3* since the first digit is *2.*
2) Count nine sections from the division marked *2,* since the second digit of the number is *9.*
3) Since the third digit is *7,* count three and one-half spaces from the left of the section. Each space has a value of two in the third digit.

Since the fourth digit is zero, it is not used in locating the number.

Numbers having more than three digits. Only the first three digits of any number which has *2* for its first digit can be represented on the C and D scales. If the number has more than three digits, those after the third do not affect the location of the number, but are treated as though they were zeros. Thus, the number *25342* would be located as *25300* on the slide rule. In any case in which the fourth digit is less than *5,* the third digit is left unchanged; in any case in which the fourth digit is *5* or greater, the third digit is increased by one in order to get a better representation of the number. For example, the number *2768* is closer to *2770* than to *2760,* so *2770* would be used in locating the number on the C or D scale.

Likewise, in the portion of the C scale to the right of the division marked *3,* only three digits of a number can be used in locating the number. If there are more than three digits, those after the third are treated as though they were zeros.

When the First Digit Is 3. Any number which has *3* for its first digit is located between the divisions marked *3* and *4*. This portion of the scale is similar to that between divisions *2* and *3*. There are ten sections between divisions *3* and *4*, and each section is divided into five spaces. Each space then has a value of two in the third digit of the number. As before, the second digit indicates in which section the number lies, and the third digit gives the number of spaces from the left end of the section. The following example will illustrate.

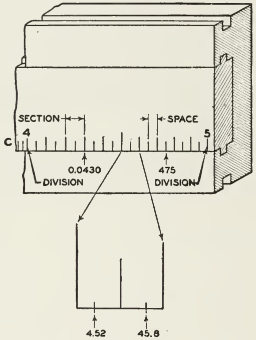


Fig. 8

ILLUSTRATIVE EXAMPLE

18. To locate 31.5.

1) Since the first digit of the number is *3*, the number is located between the divisions *3* and *4*.
2) The second digit is *1*, so count one section from division *3*.
3) From here count two and one-half spaces to locate the number. The third digit is *5* and each space has a value of two in the third digit.

When the First Digit Is 4. Fig. 8 shows the portion of the C scale between the divisions *4* and *5*. Any number which has *4* for its first digit is located here. The second digit of the number determines the proper section. The number can be located pre-

cisely only if the third digit is *5* or *0*. There are only two spaces in each section in this portion of the scale, and consequently each space must have a value of five in the third digit of the number. Correct locations for the numbers *0.0430* and *475* are shown in Fig. 8. The manner of locating them is described in the following examples.

ILLUSTRATIVE EXAMPLES

19. To locate 0.0430.

1) The first digit of the number is *4*. Hence the number is located between divisions *4* and *5*.
2) The second digit is *3*, so count three sections from division *4*.
3) Since the third digit is *0*, no spaces are counted from here. The number is located at the right end of the third section.

20. To locate 475.

1) Since the first digit of the number is *4*, the number is located between divisions *4* and *5*.
2) Count seven sections from division *4*, since the second digit is *7*.
3) From here count one space, because the third digit of the number is *5*, and each space has a value of five in the third digit.

If the third digit of the number is neither *5* nor *0*, its location may be determined approximately by estimating to the proper one-fifth of a space. Fig. 8 also shows a larger view of one section of this portion of the scale with correct locations for the numbers *4.52* and *45.8*. The following examples demonstrate the procedure used in locating them.

ILLUSTRATIVE EXAMPLES

21. To locate 4.52.

1) The first digit of the number is *4*, so the number is located between divisions *4* and *5*.
2) Since the second digit is *5*, count *5* sections from division *4*.
3) The third digit is *2*, but each space on the scale has a value of five in the third digit. Hence, estimate two-fifths of a space beyond the fifth section and locate the number here.

22. To locate 45.8.

1) The number is located between divisions *4* and *5* since the first digit of the number is *4*.
2) The second digit is *5* so count *5* sections from division *4*.
3) From here estimate one and three-fifths spaces from the fifth section. Each space has a value of five in the third digit and the third digit is *8*.

Only the first three digits of any number beginning with *4* can be represented on the C or D scale. If there are more than three digits, the remainder does not affect the location of the number. Thus the number *4832* would be set as *483*.

When the First Digit Is 5, 6, 7, 8, or 9. Marks in portions of the scale to the right of division *5* are similar to the portion between divisions *4* and *5*. The following examples will show how such numbers are located.

ILLUSTRATIVE EXAMPLES

23. To locate 504.

1) Since the first digit of the number is *5*, the number is located between divisions *5* and *6*.
2) The second digit is *0*, so the number lies in the first section of this portion of the scale.
3) The third digit is *4*, but each space has a value of five in the third digit. Hence estimate four-fifths of a space from division *5*.

24. To locate 0.66738.

1) The number is located between divisions *6* and *7* since the first digit is *6*.
2) Count 6 sections from division *6* since the second digit is *6*.
3) The third digit is *7* so estimate one and two-fifths spaces from the end of the sixth section.
4) You cannot show the fourth and fifth digits of the number so ignore them.

25. To locate 7.85.

1) The first digit of the number is *7*. Hence the number is located between divisions *7* and *8*.

2) Since the second digit is *8*, count eight sections to the right of division *7*.
3) The third digit is *5* so count exactly one space to the right from this point.

26. To locate 8190.

1) Any number beginning with *8* must be located between divisions *8* and *9*.
2) The second digit is *1* so count one section to the right from division *8*.
3) Since the third digit is *9*, estimate one and four-fifths spaces from the right of the first section. The number is located here.

27. To locate 0.00936.

1) Since the first digit is *9*, the number is located between division *9* and the right index of the scale.
2) From division *9*, count three sections since the second digit is *3*.
3) From here estimate one and one-fifth spaces. The third digit is *6* and each space has a value of five in the third digit.

THE CI SCALE

The CI scale, also called the reciprocal scale, is on the front of the slide and is marked by the letters *CI* at its left end. The CI scale can be seen in Fig. 1. It can be used with the C and D scales for multiplication and division and is especially useful for calculations which involve several operations of multiplication and division in sequence, such as, $\dfrac{47.3 \times 0.652 \times 5.3}{3.17 \times 158}$

The CI scale has exactly the same markings as the C scale but is read from right to left. Hence, one who is familiar with the C scale can locate a number readily on the CI scale. An example will show how the scale is read from right to left.

ILLUSTRATIVE EXAMPLES

28. To locate 37.7 on the CI scale.

1) The first digit of the number is *3* so the number is located between the divisions *3* and *4*.

2) Since the second digit is *7*, count seven sections to the left from division *3*.
3) From here, estimate three and one-half spaces to the left. The third digit is *7* and in this portion of the scale each space has a value of two in the third digit.

THE A AND B SCALES

The A and B scales are the identical scales which are located on the upper portion of the front of the slide rule. They are shown in Fig. 1. The A scale is on the upper part of the stock and the B scale is on the slide.

USE OF THE A AND B SCALES. The A and B scales are used with the C and D scales for finding the square, square root, cube, or cube root of a number; and also, with the sine scale for finding the sine of an angle or the arc sine of a number.

MARKING OF THE A AND B SCALES. Each scale is divided into two identical parts by the *1* at the center which is called the center index. The *1* at the extreme left of each scale is called the left index and the *1* at the extreme right is called the right index. Each half of the scale is similar to the D scale in that it offers the same numerical range. The divisions marked *2, 3, 4, 5, 6, 7, 8* and *9* occur in each half of the A scale, just as they occur in the C scale and there are still ten sections between each pair of consecutive divisions. However, because the scale has been compressed, there is a smaller number of spaces in each section than on the C scale. Between the left index and the division marked *2*, there are five spaces in each section; in the portion of the scale between the divisions marked *2* and *5*, there are two spaces in each section; and between the division marked *5* and the center index, the sections are not divided, that is, a space occupies a whole section.

HOW TO LOCATE A NUMBER ON THE A AND B SCALES. Since the right half of each scale is a duplicate of the left half, a given number could be located in either half. The question of which half is to be used in each problem is discussed in later chapters and the remarks in this section can be taken as applying to either half. The manner of locating a number is much

the same as on the C and D scales. The first digit of the number locates the proper division mark, the second digit tells you how many sections to count to the right from the division mark, and the third digit indicates the number of spaces to be counted to the right from the section mark. The only new type of marking occurs between the division marked *5* and the right end of each half of the scale. Here a space is the same as a section, so the third digit must be set approximately by estimating to the tenth of a section. The portion of the A scale between divisions *6* and *8* is shown in Fig. 9, where the numbers 0.0675 and 70.7 are shown in their correct locations. The manner of locating each is given in the examples which follow.

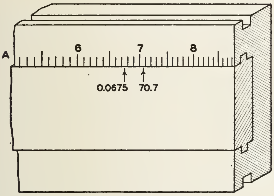


Fig. 9

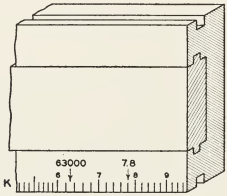


Fig. 10

ILLUSTRATIVE EXAMPLES

29. To locate 0.0675.

1) The first digit is *6* so the number lies between divisions *6* and *7*.
2) The second digit is *7*, so count *7* sections to the right from the division marked *6*.
3) From here estimate five-tenths of the section, since the third digit is *5*.

30. To locate 70.7.

1) Since the first digit is *7*, the number is located between divisions *7* and *8*.
2) The second digit is *0*, so the number lies in the first section.
3) Since the third digit is *7*, the number is located at seven-tenths of the section from the left end of the section, that is from division *7*. You must estimate the seven-tenths.

Since there are fewer spaces to each section, a number cannot be located as precisely on the A and B scales as on the C and D scales. For this reason, the use of the A and B scales for multiplication and division is not recommended, although it is possible. Only the first three digits of a number can be used in locating it on the A or B scale. The remainder must be treated as though each were zero. Thus, the number 7363 would be located the same as 7360, and in any case in which the fourth digit is less than *5*, the third digit is left unchanged. If the fourth digit is *5* or greater, the third digit is increased by one. For example, the number 5827 would be set as 5830.

THE K SCALE

The K scale is located on the lower part of the front of the slide rule. See Fig. 1.

USE OF THE K SCALE. The K scale is used with the D scale for finding the cube or cube root of a number. For this reason it is often called the cube scale.

MARKING OF THE K SCALE. The K scale is a triple scale in that it consists of three scales exactly alike, each occupying one-third of the total length. Each of the three parts is similar to the C scale, since it has the same numerical range and since it has the same division marks as the C scale. However, because the scale has been compressed, there are not as many sections and spaces between consecutive division marks as on the C scale. From the left end of each third to the division marked *3*, there are ten sections between consecutive division marks and each section is divided into two spaces. From the division marked *3* to the division marked *6*, there are ten sections between consecutive division marks but the sections are not divided. In the remainder of the scale there are only five sections between consecutive division marks.

HOW TO LOCATE A NUMBER ON THE K SCALE. The K scale consists of three similar lengths and a given number can be located in any one of the lengths. Which to use in the specific problem will be discussed later. The manner of locating a number in a particular one of the three lengths is much the

same as that of locating a number on the B or C scale. There are three types of marking on the K scale. They are:

1. Between the left end of each third and the division marked *3,* where there are ten sections between division marks and each section is divided into two spaces.

2. Between divisions *3* and *6* where there are ten sections between division marks and the sections are not divided.

3. In the remainder of the scale where there are only five sections between division marks.

The first two types of marking have been considered already in the discussions of the A, B, C, and D scales. In the portions of the scale where the third type occurs, the first digit of the number locates the number between the proper division marks. The second digit of the number determines how many sections are to be counted from the division mark. Since there are only five sections between division marks, each section must have a value of two in the second digit of the number. Hence the number can be located precisely only if the second digit is even. If it is odd, the number must be located approximately by estimating one-half of a section. Fig. 10 shows a portion of the K scale with correct locations of the numbers 63000 and 7.8. The following examples show how these numbers are located.

ILLUSTRATIVE EXAMPLES

31. To locate 63000.

1) The fact that the first digit is *6* shows that the number lies between divisions *6* and *7.*
2) The second digit is *3,* so the number is located one and one-half sections to the right of division *6.* Each section has a value of two in the second digit.

32. To locate 7.8.

1) The first digit is *7,* so the number is located between divisions *7* and *8.*
2) The second digit is *8,* so count four sections to the right from division *7.* This locates the number.

If the first digit of the number is *5* or less, the first three digits of the number can be used in locating it on the K scale. The remainder must be treated as though each were zero. If the first digit is *6* or greater, only the first two digits can be represented in locating the number on the K scale.

THE SINE SCALE

The sine scale is the top scale on the back of the slide and is designated by the letter *S* at its right end. A portion of the sine scale can be seen in Fig. 5. In order to follow this discussion, you will find it desirable to pull the slide out of the stock and turn it over, so that all of the sine scale may be seen at once.

USE OF THE SINE SCALE. The sine scale is used with the A and B scales to find the sine of an angle or the arc sine of a number. It is much quicker to find the sine of an angle from the slide rule than to look it up in a table of trigonometric functions.

MARKING OF THE SINE SCALE. You do not locate a number on the sine scale. You locate an angle in degrees and minutes because the scale is constructed for that purpose. Thus the mark on the extreme left end represents an angle, *0* degrees *34* minutes.* The mark on the extreme right end represents *90°*. Except for those at the left, specifically marked in minutes (as *50′*), all numerals printed on the scale are to be read in degrees.

In the portion of the scale to the left of the *1°* mark, each space between marks represents 2′ of angle. This portion of the scale is shown in Fig. 11. You will recall that there are 60′ in 1°. Hence there are 10′ between 50′ and 1°, represented by only five spaces.

Fig. 12 shows the portion of the sine scale between *1°* and *3°*. Here the length corresponding to one degree of angle is divided into six sections. Since there are 60′ in 1°, each section represents 10′ of angle. Each section is divided into five spaces so each space represents 2′ of angle.

Between *3°* and *10°* on the sine scale, the length representing 1° is divided into six sections, each representing 10′ of angle. Fig.

*Hereafter, the mark (°) is to be read as degrees and the mark (′) is to be read as minutes. For instance *0° 34′* is *0* degrees *34* minutes.

13 shows this portion of the scale. Each section is divided into two spaces and each space represents 5′ of angle.

Fig. 14 shows the portion of the sine scale between *10°* and *20°*. As before, the length corresponding to *1°* of angle is divided into six sections, so each section represents 10′. The sections are not divided.

In the portion of the scale between *20°* and *40°*, the length representing 1° is divided into only two sections. Hence each section represents one-half degree or 30′. This portion of the scale is shown in Fig. 15.

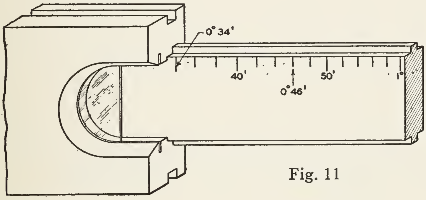


Fig. 11

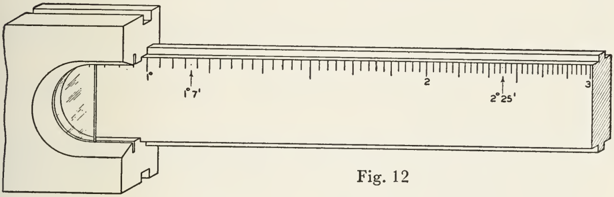


Fig. 12

The portion of the scale between *40°* and *70°* has only one space for each degree of angle. This is shown in Fig. 16.

Fig. 16 also includes the end portion of the scale to the right of *70°*. The first five spaces to the right of *70°* each represent *2°* of angle, so the fifth mark to the right of *70°* is read as *80°*. The last mark on the scale is read as *90°*.

HOW TO LOCATE AN ANGLE ON THE SINE SCALE. When the Angle Is Less Than 3°. The problem is to find the proper location for an angle which is expressed in degrees and minutes. Any angle between *0°34′* and *3°* is located in the portion of the sine scale shown in Figs. 11 and 12. The

number of degrees is considered first since it indicates the degree mark from which sections and spaces are to be counted to the right to represent the number of minutes. If the number of minutes is greater than ten, as in the angle *2°34′*, the first digit of the number of minutes shows how many sections are to be counted from the proper degree mark. If the number of minutes is less than ten, the angle is located in the first section to the right of the degree mark. There are, of course, six sections in each length that corresponds to 1°, and, since there are 60′ in 1°, each section represents 10′. Once the proper section is established, you would count further to the right, a number of spaces equal to one-half the last * digit in the number of minutes, since each space represents 2′ of angle. If the second digit of the number of minutes is odd, it would be necessary to estimate one-half of a space. Figs. 11 and 12 show correct locations for the angles *0°46′*, *1°7′* and *2°25′*. The following examples demonstrate the procedure for locating each.

ILLUSTRATIVE EXAMPLES

33. To locate 0°46′ on the sine scale.

1) The angle is less than one degree so it is located to the left of the *1°* mark.
2) *40′* and *50′* are marked on the scale. The angle lies in the section between them.
3) The last digit of the number of minutes is *6*, so count three spaces to the right from *40′*. This locates the angle. Each space represents 2′ of angle.

34. To locate 1°7′.

1) The number of degrees is *1* so the angle lies to the right of the *1°* mark on the scale.
2) The number of minutes is less than ten. Hence the angle lies in the first section to the right of the *1°* mark.
3) The last digit in the number of minutes is *7*. Each space represents 2′ of angle, so count three and one-half spaces to the right from the *1°* mark.

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*If the number of minutes is less than ten, there is only one digit in the number of minutes, and this is the last digit. There cannot be more than two digits in any number of minutes since there are only *60′* in *1°*.

35. To locate 2°25′.

1) Since the number of degrees is 2, the angle is located to the right of the *2°* mark.
2) From the *2°* mark count two sections to the right, corresponding to the first digit in the number of minutes.
3) From here count two and one-half spaces to the right to locate the angle, since each space represents *2′* of angle.

When the Angle Is between 3° and 10°. Any angle between *3°* and *10°* is located in the portion of the sine scale shown in Fig. 13. Here there is a mark for each degree, and each length corresponding to 1° is divided into six sections, each representing 10′ of angle. As before, the first digit of the number of minutes (providing the number is greater than ten) is the

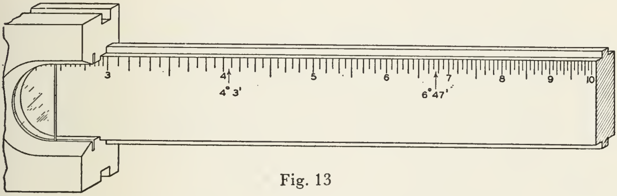

Fig. 13

number of sections to be counted to the right from the degree mark. The number of spaces to count to the right from here can be determined from the last digit in the number of minutes. Since each space represents 5′ in this portion of the scale, you count one-fifth of a space for each minute as indicated by the last digit in the number of minutes. If the number of minutes is less than ten, the angle is located in the first section to the right of the proper degree mark. Correct locations for the angles *4°3′* and *6°47′* are shown in Fig. 13. The method of locating each is shown in the following examples.

ILLUSTRATIVE EXAMPLES

36. To locate 4°3′.

1) Start with the number *4* on the scale since the number of degrees is *4*.

2) The number of minutes is less than ten, so the angle is located in the first section to the right of *4°*.
3) The last digit in the number of minutes is *3* and each space in this portion of the scale represents 5′, so estimate three-fifths of a space to the right from the *4°* mark.

37. To locate 6°47′.

1) The number of degrees is *6*. Hence, the angle is located to the right of the *6°* mark.
2) From the *6°* mark, count four sections to the right, since the first digit in the number of minutes is *4*.
3) From here count one and two-fifths spaces to the right. The last digit in the number of minutes is *7*, which is one and two-fifths of *5*. Each space represents 5′.

When the Angle Is between 10° and 20°. Fig. 14 shows the portion of the scale between *10°* and *20°*. The four longest marks between *10°* and *15°* represent, from left to right, 11°, 12°, 13°, and 14°; between *15°* and *20°*, the four longest marks represent 16°, 17°, 18° and 19°. Each length corresponding to 1° is divided into six sections, with each section representing 10′ of angle. The sections are not subdivided.

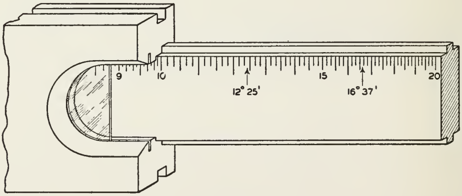

Fig. 14

In locating an angle you would start with the number of degrees. From the proper degree mark, you would count to the right a number of sections equal to the first digit of the number of minutes, if the number of minutes is greater than ten. If less than ten, the angle is located in the first section to the right of the degree mark. The last digit of the number of minutes shows

how many tenths of a section must be estimated. The angles *12°25′* and *16°37′* are shown in their proper locations in Fig. 14. The following examples demonstrate the procedure of locating each.

ILLUSTRATIVE EXAMPLES

38. To locate 12°25′.

1) Start at the mark representing *12°*. This is the second of the longest marks to the right of *10°*.
2) Count two sections to the right from *12°*, since the first digit of the number of minutes is *2*.
3) The last digit of the number of minutes is *5* so estimate five-tenths of a section more. Each section represents 10′.

39. To locate 16°37′.

1) Start with the *16°* mark which is the first of the longest marks to the right of *15°*.
2) From here count three sections to the right, since the first digit of the number of degrees is *3*.
3) From here, estimate seven-tenths of a section to the right. This locates the angle, since the last digit of the number of minutes is *7*.

When the Angle Is between 20° and 40°. The portion of the sine scale between *20°* and *40°* is shown in Fig. 15. Here,

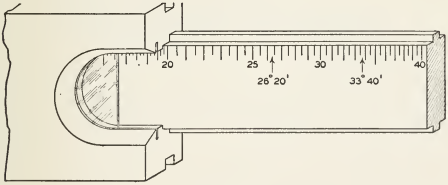

Fig. 15

the space representing 1° is divided into only two sections, so each section represents 30′ of angle. The number of degrees in the angle shows at which degree mark to start. Then the number of minutes shows how many sections to count. In this portion of the scale it is not possible to locate an angle to the nearest minute.

Hence it is best to locate the angle to the nearest 10′, or sixth part of a degree. Thus *26°22′* would be located as *26°20′*, *33°38′* would be located as *33°40′*, etc. Keeping in mind that each section represents 30′, the proper number of sections can be estimated. Fig. 15 shows the angles *26°20′* and *33°40′* in their correct locations on the scale, and the following examples show how they would be located.

ILLUSTRATIVE EXAMPLES

40. To locate 26°20′.

1) Start with the mark representing 26°.
2) From here estimate two-thirds of a section to the right. (Twenty is two-thirds of thirty, hence 20′ is represented by two-thirds of a section since a section represents 30′.)

41. To locate 33°40′.

1) Start with the mark representing *33°*.
2) From here estimate one and one-third sections for 40′.

When the Angle Is between 40° and 70°. Fig. 16 shows the portion of the sine scale to the right of *40°*. In this portion, each space represents 1°. Thus it is not possible to locate an angle

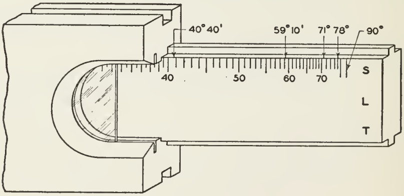

Fig. 16

to the exact number of minutes. Instead, the angle should be "rounded off" to the nearest 10′, which is one-sixth of a degree. The number of degrees in the angle indicates the proper degree mark from which to start and then the correct fraction of a space to correspond with the number of minutes is estimated. The angles *40°40′* and *59°10′* are shown in their proper locations in

Fig. 16. The following examples demonstrate the method for locating them.

ILLUSTRATIVE EXAMPLES

42. To locate 40°40′.

1) Start with the *40°* mark.
2) From here estimate two-thirds of a space to the right; *40′* is two-thirds of *60′*, which is *1°*, and each space represents 1°.

43. To locate 59°10′.

1) Start with the *59°* mark, which is one space to the left of the *60°* mark.
2) From here, estimate to the right, one-sixth of a space, since 10′ is one-sixth of 1°.

When the Angle Is between 70° and 80°. Fig. 16 also shows the portion of the sine scale to the right of *70°*. Each of the first five spaces to the right of *70°* represents 2° of angle. Thus the mark next to the last mark represents 80°. The last mark on the scale represents *90°*. In this portion of the scale it is best to locate the angle only to the nearest degree. Even this requires estimating half a space if the number of degrees is odd. Fig. 16 also shows the angles *71°* and *78°* in their correct locations on the sine scale. The procedure of locating each follows.

ILLUSTRATIVE EXAMPLES

44. To locate 71°.

1) Estimate one-half space to the right from the *70°* mark. Each space represents 2°; consequently, *71°* is one-half space to the right from *70°*.

45. To locate 78°.

1) 78° is 8° greater than 70°; hence, count four spaces to the right from the *70°* mark.

Angles between 80° and 90°. It is not feasible to locate angles between 80° and 90° on the sine scale. Fortunately, it is not necessary to do so, since there is a method by which the sine of such a large angle can be found without actually setting

the angle on the sine scale. This method is discussed in the chapter on sines and cosines.

Angles Less Than 0°34′. An angle less than 0°34′ cannot be located on the sine scale. However, the sine of such a small angle can be found from the slide rule. A suitable method is discussed in the chapter on sines and cosines.

THE TANGENT SCALE

The tangent scale is the bottom scale on the back of the slide. It is designated by the letter *T* at its right end. A portion of the tangent scale can be seen in Fig. 5.

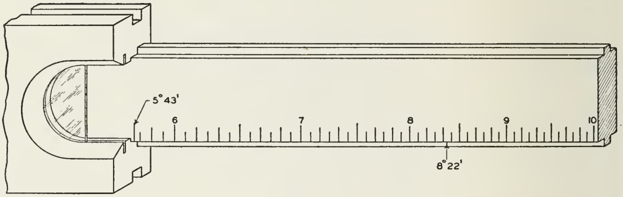

Fig. 17

USE OF THE TANGENT SCALE. The tangent scale is used with the C and D scales to find the tangent of an angle or the arc tangent of a number. The time required for one of these operations on the slide rule is much less than that required to look up the tangent in a table of trigonometric functions.

MARKING OF THE TANGENT SCALE. The mark at the extreme left end of the tangent scale represents *5°43′*. The mark at the extreme right end represents *45°*. All numbers printed on the scale are read in degrees. There are only two types of marking on the scale, one type to the left of *20°* and the other to the right of *20°*. A portion of the left end is shown in Fig. 17. Here you will notice that the length corresponding to 1° is divided into six sections, each representing 10′, and each section is divided into two spaces, each representing 5′. There are, of course, 60′ in 1°.

Fig. 18 shows a portion of the right end of the tangent scale. In this portion, 1° is divided into six sections, each representing 10′. The sections are not subdivided.

HOW TO LOCATE AN ANGLE ON THE TANGENT SCALE. When the Angle Is between 5°43′ and 20°. Much of what has been said of the sine scale applies here. The problem is still the same, that of locating on the scale an angle which is expressed in degrees and minutes. The number of degrees shows the degree mark from which to start counting sections and spaces to the right. If the number of minutes is greater than ten, the first digit of the number tells how many sections to count to the right from the degree mark. If the number of minutes is less than ten, the angle is located in the first section to the right of the degree mark. The last digit of the number of minutes determines how many one-fifths of a space are to be counted in order to locate the number, since each space represents 5′ of angle. In Fig. 17 is shown the correct location for the angle *8°22′*. The following example shows the procedure of locating it.

ILLUSTRATIVE EXAMPLE

46. To locate 8°22′ on the tangent scale.

1) Start with the *8°* mark.
2) The first digit of the number of minutes is *2*, so count two sections to the right from *8°*.
3) From here, estimate two-fifths of a space to the right, since the last digit of the number of minutes is *2*.

When the Angle Is between 20° and 45°. The number of degrees gives the degree mark from which to count sections to the right. If the number of minutes is greater than ten, the first digit of the number is the number of sections to be counted to the right. If the number of digits is less than ten, the angle is located in the first section to the right of the degree mark. Each section is undivided and represents 10′ of angle, so the last digit of the number gives the number of tenths of a section to be estimated. Fig. 18 shows the correct location for the angle *33°47′*. The steps in the process of locating it are given in the following example.

ILLUSTRATIVE EXAMPLE

47. To locate 33°47′.

1) Start with *33°* which is the third of the longest marks to the right of *30°*.
2) Count four sections to the right of *33°* since the first digit of the number of minutes is *4*.
3) From here estimate seven-tenths of a section to the right. This locates the angle. The last digit of the number of minutes is 7, and since each section represents 10′, this represents seven-tenths of a section.

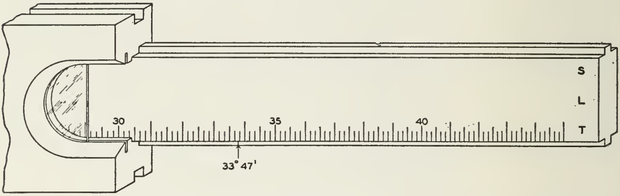

Fig. 18

Angles Less Than 5°43′. Angles less than *5°43′* need not be located on the tangent scale. Other methods exist for finding the tangent of such a small angle and these methods are discussed in the chapter on sines and cosines.

Angles Greater Than 45°. The tangent of an angle greater than 45° can be found by locating on the tangent scale an angle less than 45°. The procedure is discussed in the chapter on sines and cosines.

THE LOG SCALE

LOGARITHMS TO THE BASE 10. Although the slide rule is based on logarithms, no knowledge of logarithms is necessary for its successful operation. You can become very proficient in its use without knowing anything of logarithms. However, there are many problems in advanced work which require the use of

logarithms in calculation, problems in such subjects as Analytical Mechanics, Thermodynamics and Fluid Mechanics. Typical examples include the calculation of belt friction, vibration phenomena, and heat transfer. One who wishes to do such work must have a knowledge of logarithms. It is convenient in working problems of this kind to be able to obtain the logarithm of a number from the slide rule so that it is not necessary to consult a table of logarithms.

The logarithm of a number to the base 10 is the power to which 10 must be raised to equal the number. The logarithm of *100* to the base 10 is *2.000* since 10 must be raised to the power *2.000* to equal *100*. The logarithm of *348* to the base 10 is *2.542* since 10 must be raised to the power *2.542* to equal *348*.

Mantissa of a Logarithm. The part of the logarithm to the right of the decimal point is called the *mantissa*. For the logarithm *2.542,* the mantissa is *542*. The base 10 is convenient because the mantissa of the logarithm of a number to the base 10 is the same regardless of the location of the decimal point of the number. Thus the mantissa is *542* for the numbers *0.0348, 3.48, 348, 34800,* etc. This fact is used to advantage in constructing the scales of the slide rule in that the distance from the left end of the scale to a number is laid off, not as the number itself, but as the mantissa of the logarithm of the number to the base 10. Each of the foregoing numbers then, occupies the same position on the scale of the slide rule.

Characteristic of a Logarithm. The part of the logarithm to the left of the decimal point is called the *characteristic*. Thus, since the logarithm of *348* is *2.542,* the characteristic is *2*. The characteristic of the logarithm of a number to the base 10 is one less than the number of digits to the left of the decimal point of the number when the number is greater than 1. For example, since there are three digits to the left of the decimal point in the number *348,* the characteristic of its logarithm is one less than three, or *2*. The rule may be applied to numbers less than one as follows: (a) For a decimal fraction having no zeros immediately to the right of the decimal point, the characteristic of its logarithm is one less than zero, or -1. For example, the

characteristic of the logarithm for *.432* is zero minus one, or *—1.* (b) For decimal fractions having one or more zeros immediately to the right of the decimal point, each such zero is regarded as a negative digit. For example, the characteristic of the logarithm for *0.00047* is one less than minus three, or *—4* *. The following table may be found useful in determining the characteristic of the logarithm of any number to the base 10.

TABLE 1—CHARACTERISTICS

Numbers Between			Calculation of Characteristic: No. of Digits	Less 1		Characteristic
100,000	and	1,000,000	6	—1	=	5
10,000	and	100,000	5	—1	=	4
1,000	and	10,000	4	—1	=	3
100	and	1,000	3	—1	=	2
10	and	100	2	—1	=	1
1	and	10	1	—1	=	0
0.1	and	1	0	—1	=	—1†
0.01	and	0.1	—1	—1	=	—2
0.001	and	0.01	—2	—1	=	—3
0.0001	and	0.001	—3	—1	=	—4
0.00001	and	0.0001	—4	—1	=	—5

1. *If the number is greater than 1 the characteristic is one less than the number of digits to the left of the decimal point (its digit count) and is always positive.*

2. *If the number is less than 1 the characteristic is one more than the number of zeros immediately to the right of the decimal point (its digit count), and is always negative.*

USE OF THE LOG SCALE. The log scale is the center scale on the back of the slide. It is designated by the letter *L* at the right end. A portion of the log scale can be seen in Fig. 5.

The scale is used with the C and D scales to find the mantissa of the logarithm of a number to the base 10, or to find the number which has a certain mantissa. Only the mantissa needs to be determined with the slide rule since the characteristic is determined

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*An explanation of negative numbers is given on page 240.

†— 1 is read as *minus 1,* a negative number.

either by the number of digits to the left of the decimal point in the number, or by the number of zeros to the right of the decimal.

MARKING OF THE LOG SCALE. The log scale is the simplest of all in its marking. It is divided into ten equal lengths and each of these lengths is divided into ten sections. Each section is subdivided into five spaces.

HOW TO LOCATE A NUMBER ON THE LOG SCALE. The first numeral (it may be from *0* to *9*) of the number determines the particular length in which the number is located. The second numeral determines the particular section

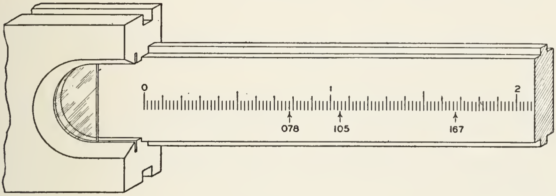

Fig. 19

in this length. The third numeral determines how many spaces are to be counted. The division of the log scale is such that only the first three numerals of the number can be used in locating it. Since there are only five spaces in each section, each space must have a value of two in the third numeral. Consequently, the number can be located exactly only if the third numeral is even. If it is odd, the number will be located in the center of a space, which means that it is necessary to estimate one-half of a space. Fig. 19 shows a portion of the log scale with correct locations for the numbers 078, 105, and 167. The procedure for locating each is given in the following examples.

ILLUSTRATIVE EXAMPLES

48. To locate 078 on the log scale.

1) The first numeral is *0,* so the number is located to the left of the number *1* printed on the scale.

2) The second numeral is *7,* so count seven sections from the left end of the scale.
3) From here count four spaces to the right. The third numeral is *8,* but each space has a value of two in the third numeral.

49. To locate 105.

1) Start at the number *1* printed on the scale since the first numeral of the number is *1.*
2) The second numeral is *0* so the number is located in the section immediately to the right of 1.
3) From the printed number *1,* count two and one-half spaces to the right. Each space has a value of two in the third numeral and the third numeral is *5.*

50. To locate 167.

1) Start at the number *1* printed on the scale since the first numeral of the number is *1.*
2) The second numeral is *6* so count six sections to the right.
3) Count farther, three and one-half spaces, since the third numeral is *7,* and each space has a value of two in the third numeral.

REVIEW QUESTIONS

If you have studied the material of this chapter carefully, you should be able to answer the following questions.

1. What is a scale on the slide rule?

2. Where on the slide rule is the left index of the D scale?

3. What is the first digit of a number?

4. Which digits of a number can be zero?

5. What is a section? What is a space?

6. Which is larger, a section or a space?

7. Would the numbers *1.457* and *145.7* be located the same on the C scale?

8. On the C scale, which digit of a number determines the section in which the number lies, and which digit determines the space in which it lies?

9. How many digits of the number *19.3721* can be represented in locating the number on the D scale?

10. Can any number, no matter what its size, be located on the D scale?

11. What is the center index on the B scale?

12. How many digits of the number *978.42* can be used in locating the number on the B scale?

13. What is a division mark on the C scale?

14. On which scales of the slide rule do you locate a number and on which scales do you locate an angle?

15. Can the same number be located in more than one place on the C scale?

16. In what order do you consider the digits of a number when you locate it on the A scale?

17. How many minutes are there in an angle of 1°?

18. Which scale reads from right to left?

19. Which digit of the number do the printed numerals on the A scale represent?

20. What is the largest angle that can be set on the sine scale?

21. What is the largest angle that can be set on the tangent scale?

CHAPTER IV

MULTIPLICATION

THE PROCESS OF MULTIPLICATION. Multiplication is ordinarily accomplished with the C and D scales. The simplest operation is that of multiplying one number, the *multiplicand,* by another number, the *multiplier.* The process is carried out in the following steps:

STEPS IN THE PROCESS OF MULTIPLICATION

1) The first number, or multiplicand, is located on the D scale by adjusting the slide so that one index of the C scale coincides with the number.
2) The runner is then placed in such a position that the hairline coincides with the second number, or multiplier, on the C scale.
3) The answer is read on the D scale under the hairline. These steps can be stated as a rule. It is:

> **Rule 1. Multiplication.** *Set one index of the C scale to the multiplicand on the D scale. Next, set the hairline of the runner to the multiplier on the C scale. Finally, read the answer on the D scale under the hairline.*

ILLUSTRATIVE EXAMPLES

Read the following examples carefully, and check each operation on your own slide rule.

1. To multiply 11.7×136. Fig. 20 shows the correct positions of the slide and runner for this operation. Only the C and D scales are shown in detail, since they are the only ones used in this multiplication. The operation should be performed in the following steps:

1) Move the slide until the left index of the C scale is over the number *11.7,* the multiplicand, on the D scale.
2) Place the runner so that the hairline coincides with the number *136,* the multiplier, on the C scale.
3) Read the answer, *1591,* on the D scale under the hairline*.

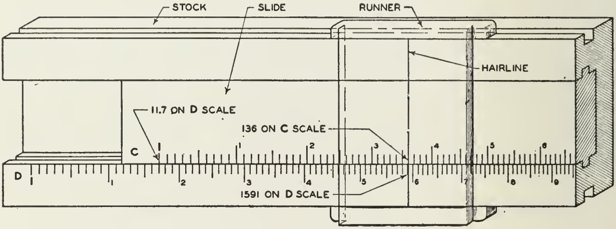

Fig. 20

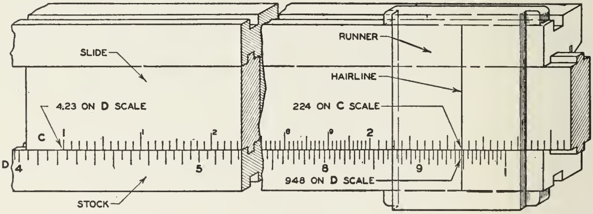

Fig. 21

2. To multiply 4.23 × 224. The correct positions for the slide and runner are shown in Fig. 21. The steps in the process are:

1) Set the left index of the C scale to *4.23* on the D scale.
2) Place the hairline of the runner on *224* on the C scale.

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*It is necessary to estimate one-tenth of a space in order to determine the fourth digit in this answer. Slight inaccuracies in setting the multiplier, *136,* and the multiplicand, *11.7,* might cause you to read the answer as *1590* or *1592*. However, the probable error is limited to one point in the fourth digit in this example. There are very few problems in which such an error would be serious.

3) Read the answer, *948,* on the D scale under the hairline. Here of course, the third digit is estimated, but the maximum error, with careful work, is one point in the third digit.

Not all multiplications can be performed by starting with the left index of the C scale and moving the slide to the right. In many cases the multiplier on the C scale would be beyond the right end of the stock. You can demonstrate by trial that this is true for the multiplication *72 × 93.* However, in any case in which the operation cannot be completed by starting with the left index of the C scale, it can be accomplished by starting with

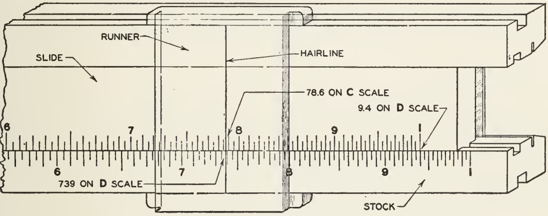

Fig. 22

the right index of the C scale and moving the slide to the left. The following examples demonstrate this.

ILLUSTRATIVE EXAMPLES

3. To multiply 9.4 × 78.6. Fig. 22 shows the correct position of the slide and runner for this calculation. The procedure is:

1) Place the right index of the C scale on *9.4* on the D scale.
2) Set the hairline of the runner on *78.6* on the C scale.
3) Read the answer, *739,* on the D scale under the hairline.

4. To multiply 834 × 1.55. The steps in the process are as follows:

1) This multiplication must be performed with the slide extending to the left, so place the right index of the C scale on the number *834* on the D scale.
2) Set the hairline of the runner on *1.55* on the C scale.

3) Read the answer, *1293,* on the D scale under the hairline.

5. To multiply 49.9 × 20.2. The steps in this operation are described below:

1) A trial will show that the slide must move to the left. Hence, place the right index of the C scale on *49.9* on the D scale.
2) Set the hairline of the runner on *20.2* on the C scale.
3) Read the answer *1005* on the D scale under the hairline.

Any multiplication can be performed on the slide rule, but there is never a choice as to whether to move the slide to right or left.* Only one will work. The judgment of the experienced operator will indicate which to use in most cases; in the remainder, the fact that the multiplier is located beyond the end of the stock on the C scale is a definite indication that the slide must go the other way.

Multiplication can be done in any order; it makes no difference which factor is the multiplicand and which the multiplier. However, the runner will go in the same direction no matter which order of multiplication is used.

PRACTICE PROBLEMS

1. Do each of the foregoing examples by reversing the order, that is, interchange the multiplicand and multiplier.

Solve all of the following problems; (*a*) using the first number as the multiplicand; (*b*), using the second number as the multiplicand. Then check your answers with those given in the back of the book.

2. 159 × 5.8 = ?
3. 243 × 1.97 = ?
4. 3.3 × 296 = ?
5. 31.7 × 32.2 = ?
6. 10.3 × 95.5 = ?

7. What is 27 per cent of 1680? (*Hint:* 27 per cent of a number is 27 hundredths of the number.)

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*There is one exception to this rule. If the product of two numbers is a number of which the first digit is *1* and all other digits are zero, the multiplication can be made either way. Examples of such numbers are *1, 10, 100,* and *1000.*

8. How many square feet are contained in a rectangular room 18.3 feet wide and 85 feet long?

9. A carpenter drives screws at the rate of 87 per hour. How many does he drive in a 40-hour week?

10. How far will an automobile traveling at 68 miles per hour go in 3.75 hours?

11. The operator of a turret lathe can finish 33 machine parts in one hour. How many can he finish in an 8-hour shift?

12. Steel weighs 0.283 pounds per cubic inch. How much does a cubic foot weigh? (1 cu. ft. = 1728 cu. in.)

13. A certain type of garden hose costs 12 cents per foot. How much will 66.7 feet cost?

14. A steel bar is to be loaded at the rate of 1600 pounds per square inch. How much will a bar having an area of 0.785 square inches carry?

15. A certain type of bearing requires 1.45 pounds of bearing metal. How much will 540 bearings require?

When you have reached this point, you know how to multiply one number by another. Next you should work many problems. Make up your own, or if problems arise in your work or study, work them with the slide rule. This will give practice in placing the index of the C scale at the proper number on the D scale, and in setting the hairline of the runner on a number, with the result that you will be able to perform these operations quickly and with confidence. You will find that you can multiply one number by another very rapidly and that the motions become automatic so that very little mental effort is required. The more you use the slide rule, the more effective it will be as a tool.

Problem 6, in the foregoing list, can be used to illustrate a point in connection with multiplication. When the multiplication is performed with *10.3* as the multiplicand, nearly all of the slide remains within the stock. This makes for precision since the stock holds the slide in alignment and keeps it from wobbling. When *95.5* is used as the multiplicand, nearly all of the slide projects from the stock. In such a position the slide is loose in the stock and is liable to move while the runner is brought into position, thus causing an error. This will be easy to understand if you set

your own slide rule in the positions described. In deciding which number to use for the multiplicand it is best to select the one which will cause most of the slide to remain within the stock.

ILLUSTRATIVE EXAMPLES

6. To multiply 86.6 × 11.5.

1) Try this with *86.6* as the multiplicand.
2) Try it with *11.5* as the multiplicand. Notice how much easier it is this way and how the slide is held firmly in the stock.
3) The answer is *996.*

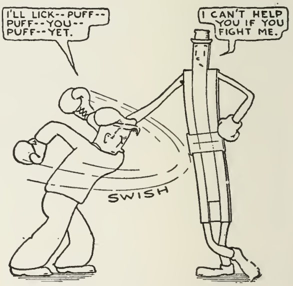
7. To multiply 9.8 × 103.

1) Try it with *103* as the multiplicand.
2) Try it with *9.8* as the multiplicand. Notice that the stock holds the slide firmly.
   The slide cannot wobble and is not likely to slip out of position while the runner is being adjusted.
3) The answer is *1009.*

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

1. 1.001 × 998 = ?
2. 940 × 1.05 = ?
3. 106.3 × 9.65 = ?
4. 12.56 × 88.3 = ?
5. 8.56 × 130.5 = ?



DON'T FIGHT THE SLIDE RULE

LOCATING THE DECIMAL POINT IN THE ANSWER. DIGIT COUNT. For purposes of this book, and to simplify the placing of the decimal point after use of the slide rule, the decimal point is located in the answer by means of the *digit count.*

The digit count for 1 or a number greater than 1 is the number of digits to the left of the decimal point in the number. This means that for 1 or for numbers greater than 1 each digit left

of the decimal point is considered a plus, or positive, digit; any numerals to the right of the decimal point are ignored. For example, in the number *672.94* there are three positive digits, hence the digit count is three (+3); those numerals to the right of the decimal point are ignored. When the number is a decimal fraction, there will never be digits to the left of the decimal point. If the first digit of such a number is immediately to the right of the decimal point, there are zero digits to the left of the decimal point and the digit count is zero; for example, the digit count for *0.612* is zero.

The *digit count* for any number less than 0.1 is a negative number, and is numerically equal to the number of zeros at the right of the decimal point and between the decimal point and the first digit of the number. Each such zero is, for purposes of this book, counted as a minus, or negative, digit. For example, for the number *0.036* the digit count is one negative digit, or minus one (−1); for *0.00061* the digit count is three negative digits, or minus three (−3).

The following table is presented to help you understand how to make the digit count in the process of locating the decimal point.

TABLE 2—DIGIT COUNTS

Number	Digit Count
400,000.	+6
40,000.	+5
4,000.	+4
400.	+3
40.	+2
4.	+1
.1	0
.01	−1
.001	−2
.0001	−3
.00001	−4
.000001	−5

Remember that all numerals are assumed to be positive when no sign is used, therefore the plus sign is not always used.

Two positive rules can be stated for locating the decimal point in the answer when multiplying one number by another.

1. Multiplication. *When the slide extends to the left, the sum of the digit count for the multiplicand and the digit count of the multiplier equals the digit count for the answer.*

If the digit count for the answer is plus, you will know it indicates the number of places to the left of the decimal point in the answer, because you will remember that in making the digit count each digit to the left of the decimal is counted as plus.

If the digit count for the answer is zero, you will know that there are no digits to the left of the decimal point in the answer and that the first digit is immediately to the right of the decimal.

If the digit count for the answer is a minus number, you will know that there are that number of zeros immediately to the right of the decimal point.

ILLUSTRATIVE EXAMPLES

8. To multiply 97.3×84.5. The sequence of numbers in the answer is read as *823*. There are two digits to the left of the decimal point in each number, or a digit count of two, giving a total of four for the two numbers. Since the slide of the rule projects to the left of the stock, the digit count in the answer is the same as this sum, namely four $(+4)$. This means that in the answer there will be four digits to the left of the decimal point. Hence a zero is added to the sequence of numbers to make it four digits, they are placed to the left of the decimal point, and the answer is *8230*.

9. To multiply $0.866 \times 19{,}700$. The sequence of numbers in the answer is read as *1707*. There are zero digits to the left of the decimal point in *0.866,* making a digit count of zero, and five digits to the left of the decimal point in *19700,* making a digit count of $+5$. The sum of zero and five is five. The slide of the rule extends to the left of the stock so the digit count for the answer is $+5$; there are five digits, then, to the left of the decimal point in the answer and it is *17,070*.

10. To multiply 0.0635 × 0.00217. The sequence of numbers in the answer is read as *1378*. The digit count for *0.0635* is minus one, and the count is minus two for *0.00217*. The sum of minus one and minus two is minus three. Since the slide projects to the left of the stock in this operation, the digit count for the answer is equal to this sum, minus three (−3). Therefore we know there are three zeros to the right of the decimal and the answer is *0.0001378*.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

1. 0.00475 × 0.0730 = ?

2. 18300 × 0.0276 = ?

3. 9.75 × 0.000540 = ?

4. 0.0172 × 863 = ?

5. 6280 × 356 = ?

6. What is 8.7% of 0.397?

7. An automobile travels at the rate of 73 feet per second. How far does it go in 92 seconds?

8. One gallon contains 231 cubic inches. How many cubic inches in 8.33 gallons?

9. Sand weighs 125 pounds per cubic foot. How much would 93 cubic feet of sand weigh?

10. One foot is equal to 12 inches. How many inches in one mile? One mile is equal to 5280 feet.

11. What is 83% of 952?

12. A certain machine gun can fire 350 bullets per minute. How many can it fire in 7.5 minutes?

2. Multiplication. *When the slide extends to the right, subtract one from the sum of the digit count for the multiplicand and the digit count for the multiplier to get the digit count for the answer.*

Remember that if the digit count for the answer is plus, it indicates the number of digits to the left of the decimal point in the answer; if it is zero, the first digit of the answer is immediately to the right of the decimal point; if it is minus, it gives the number of zeros immediately to the right of the decimal point.

ILLUSTRATIVE EXAMPLES

11. To multiply 12.6×0.318. The sequence of numbers in the answer is read as *401*. There are two digits to the left of the decimal point in the multiplicand, *12.6,* making a digit count of plus two, and zero digits to the left of the decimal point in the multiplier, *0.318,* making a digit count of zero. The sum of two and zero is two. Since the slide extends to the right of the stock in this operation, the digit count for the answer is one less than two, or one. Hence, the answer is *4.01*.

12. To multiply 2120×384. The sequence of numbers in the answer is read as *814*. The digit count is $+4$ for *2120** and $+3$ for *384*. The sum of four and three is seven. The slide projects to the right of the stock in this operation, so the digit count for the answer is one less than seven, or six. Hence, the answer is *814,000*.

13. To multiply 0.000242×0.00308. The sequence of numbers in the answer is read as *745*. The digit count is minus three for *0.000242* and minus two for *0.00308*. The sum of minus three and minus two is minus five. Since the slide extends to the right, the number of negative digits in the answer is one less than minus five, or minus six: † $-5-(+1)=-6$. Hence, the answer has six zeros immediately following the decimal point and is *0.000000745*.

14. To multiply 1240×0.000473. The sequence of numbers in the answer is read as *586*. Since there are four digits to the left of the decimal point in *1240* the digit count is four, and it is minus three for *0.000473*. The sum of plus four and minus three is one, $4+(-3)=1$. Since the slide projects to the right of the stock, the digit count for the answer is one less than one, or zero. Hence, the answer is 0.586.

---

*When a number is written without any decimal point, as *2120,* it is always understood that the decimal point follows the last digit. Here the decimal point follows the zero. The number could have been written as *2120.00*.

†A brief review of addition and subtraction with negative numbers is given on p. 240.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

1. $33000 \times 0.0000216 = ?$

2. $0.0195 \times 0.456 = ?$

3. $1.008 \times 93.4 = ?$

4. $13.3 \times 0.00067 = ?$

5. $0.0045 \times 0.0208 = ?$

6. The unit elongation of a bar of steel, for a rise in temperature of 1° F., is 0.0000065. What is the unit elongation for a rise in temperature of 139° F.?

7. A certain automobile requires 0.047 gallons of gasoline per mile. How much gasoline will it use in 114 miles?

8. A certain casting requires 218 pounds of malleable iron. How much will 37 such castings require?

9. An acre of ground is equivalent to 43560 square feet. How many square feet are contained in 0.017 acre?

10. How many square feet are there in 220 acres?

MULTIPLICATION OF EACH OF MANY NUMBERS BY ONE NUMBER. In calculating percentages, compiling tables, and similar operations, it is often necessary to multiply each of many numbers by a single number. Much time can be saved in such problems by using the single number as the multiplicand in each separate multiplication. In this way the slide needs to be set only once and can then remain fixed in the same position for all of the multipliers. The rules previously given for locating the decimal point hold here.

ILLUSTRATIVE EXAMPLE

15. Compute 82% of each of the following numbers: 983, 764, 712, 636, 603, 577, 432, 333. Since 82 per cent of a number is equal to *0.82* times the number, each number in the list must be multiplied by *0.82*. In each case the slide will project to the left of the stock. To start then, the slide is placed so that the right index of the C scale is at *82* on the D scale. The runner is placed so that its hairline is in turn at each of the numbers *983, 764, 712,* etc. on the C scale. For each the answer is read on the D scale under the

hairline. During all of the operation the slide remains fixed. The answers are, respectively: 805; 626; 584; 522; 494; 473; 354; 273.

In many such problems, part of the multiplications will require the slide to move to the right and the remainder will require the slide to move to the left. This can be treated as two separate problems, one including the multiplications which require the slide to move to the right and the other including those which require the slide to move to the left. The rules given previously for locating the decimal point hold here.

ILLUSTRATIVE EXAMPLES

16. Multiply each of the following numbers by 5.23: 1.04, 1.69, 1.85, 2.76, 3.58, 4.65.

1) Place the left index of the C scale on *5.23* on the D scale. Set the runner so that the hairline is in turn on *1.04, 1.69,* and *1.85* on the C scale. Read the answers on the D scale under the hairline, respectively: 5.44; 8.84; 9.68.
2) The rest of the problem must be done with the slide extending to the left. Place the right index of the C scale on *5.23* on the D scale. Set the runner so that the hairline is in turn on *2.76, 3.58* and *4.65* on the C scale. Read the answers on the D scale under the hairline, respectively: 14.44; 18.72; 24.3.

17. Aluminum weighs 165 pounds per cubic foot. Calculate the weight of each of the following volumes of aluminum. Each volume is given in cubic feet. 0.12, 0.375, 0.540, 0.648, 0.813, 0.950.

1) Place the left index of the C scale on *165* on the D scale. Set the hairline of the runner successively on *0.12, 0.375* and *0.540* on the C scale. Read the answers on the D scale under the hairline, respectively: 19.8 lb.; 61.8 lb.; 89.1 lb.
2) Place the right index of the C scale on *165* on the D scale. Set the hairline of the runner on *0.648, 0.813* and *0.950* in turn on the C scale. Read the answers on the D scale under the hairline, respectively: 106.9 lb.; 134.1 lb.; 156.8 lb.

PRACTICE PROBLEMS

After you have worked all of the problems, check your answers with the correct answers shown at the back of the book.

1. Calculate 89.5% of each of the following numbers: 0.915, 0.812, 0.767, 0.663, 0.533.

2. Convert each of the following volumes in cu. ft. into the equivalent volume in cu. in. 1.65, 3.76, 10.6, 15.8, 23.2, 39.5, 102.5, 133. (*Hint:* 1 cu. ft. = 1728 cu. in. so each number must be multiplied by 1728.)

3. The wholesale price for each of a number of articles is, respectively: $0.83, $0.96, $1.08, $1.17, $1.35, $1.62. The retail price for each is to be 120% of the wholesale price. Calculate the retail price of each to the nearest cent.

4. A speed of one mile per hour is equivalent to 1.467 feet per second. Convert each of the following speeds in miles per hour into feet per second. 22, 29, 30, 45, 50, 60, 72, 85, 95.

5. A series of circles has the following diameters, in feet: 35, 68.5, 104.5, 163, 216, 385. Calculate the circumference of each. (*Hint:* The circumference of a circle is 3.14 times the diameter.)

6. A bar of steel 1 inch square and one foot long weighs 3.4 pounds. Calculate the weight of each of the following lengths of 1 inch square steel bar: 0.78, 1.043, 2.17, 3.56, 5.22, 6.63, 8.2. Each length is given in feet.

MULTIPLICATION OF THREE OR MORE NUMBERS. There are many occasions on which it is necessary to multiply three or more numbers. In such cases, the operation can be carried all the way through without writing down any intermediate results. The first number is set on the D scale by placing the proper index of the C scale in coincidence with it. This is multiplied by the second number by setting the hairline of the runner to the second number on the C scale. This locates the product of the first two numbers on the D scale under the hairline. This product is then multiplied by the third number. In this process, the runner is left in position while the proper index of the C scale is brought into coincidence with the hairline. Then the runner is placed so that the hairline coincides with the third number on the C scale. The final answer is read on the D scale under the hairline.

Obviously, this process could be extended to any number of multiplications.

In order to locate the decimal point in the answer, the digit counts for the original numbers are added. For each part of the operation in which the slide extends to the right of the stock, the number one is subtracted from this sum. The resulting number is the digit count for the answer. You may find it convenient to make a mark on the paper for each part of the operation in which the slide extends to the right of the stock. The number of marks can then be subtracted from the sum of the digit counts for the multiplicand and multipliers to give the digit count for the answer.

ILLUSTRATIVE EXAMPLES

18. Multiply $157 \times 32.3 \times 0.636$. The operation is performed in the following steps:

1) Place the left index of the C scale on *157* on the D scale. Note that the slide extends to the right of the stock.
2) Set the hairline of the runner on *32.3* on the C scale. This locates the product of the first two numbers on the D scale under the hairline.
3) Leave the runner fixed and move the right index of the C scale until it is under the hairline. Note that the slide moves to the left beyond the stock.
4) Place the hairline of the runner on *0.636* on the C scale.
5) Read the final answer as the sequence of numbers *323* on the D scale under the hairline.
6) Add:

Digit count for	*157*	= 3
Digit count for	*32.3*	= 2
Digit count for	*0.636*	= 0
Sum		= 5

Since the slide of the rule extended to the right of the stock once during the operation, subtract one from five, leaving four. The digit count is + 4, hence there are four digits to the left of the decimal point in the final answer, and the answer is *3230*.

19. Multiply $0.236 \times 1.93 \times 0.00205$. The process is as follows:

1) Set the left index of the C scale to *0.236* on the D scale.
2) Place the hairline of the runner on *1.93* on the C scale, noting that the slide moves to the right of the stock.
3) Leave the runner fixed while placing the left index of the C scale under the hairline. Note that the slide again projects to the right of the stock.
4) Move the hairline of the runner to *0.00205* on the C scale.
5) Read *934* as the sequence of numbers in the final answer on the D scale under the hairline.
6) Add:

Digit count for *0.236*	=	0
Digit count for *1.93*	=	1
Digit count for *0.00205*	=	− 2
Sum	=	− 1

The slide extended to the right of the stock twice during the operation, so subtract two from minus one, leaving minus three, $-1-(+2)=-3$, the digit count for the answer. This means there are three zeros to the right of the decimal point in the final answer, which is *0.000934*.

20. Multiply $938 \times 0.775 \times 0.462$. The multiplication is performed in the following steps:
1) Place the right index of the C scale on *938* on the D scale. Note that the slide extends to the left of the stock.
2) Place the hairline of the runner on *0.775* on the C scale.
3) Leave the runner in place while moving the right index of the C scale into coincidence with the hairline. Note that the slide projects to the left of the stock.
4) Move the hairline to *0.462* on the C scale.
5) Read the sequence of numbers in the final answer as *336* on the D scale under the hairline.
6) Add:

Digit count for *938*	=	3
Digit count for *0.775*	=	0
Digit count for *0.462*	=	0
Sum	=	3

The slide did not extend to the right of the stock at all during the operation so there is nothing to be subtracted from *3*. Hence the digit count for the answer is 3. There are three digits to the left of the decimal point in the final answer and it is *336*.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown at the back of the book.

1. $0.0000123 \times 248 \times 12 = ?$ **3.** $0.866 \times 685 \times 3.2 = ?$

2. $45.4 \times 0.495 \times 356 = ?$ **4.** $3.14 \times 1.76 \times 0.93 = ?$

5. $790 \times 12.3 \times 0.707 = ?$

6. A room is 18.3 ft. long, 12.5 ft. wide and 8.2 ft. high. How many cubic ft. does it contain?

7. An automatic screw machine can make 6 fittings per second. How many can it make in 8.3 minutes?

8. A certain type of heavy truck uses 0.187 gallons of gasoline per mile traveled. How much gasoline will a fleet of 16 trucks use in traveling 42 miles?

9. What is 37 per cent of 728×0.0055?

10. The bucket of a steam shovel has a capacity of 1.4 cubic yards. The shovel can lift 38 bucketfuls of dirt in one hour. How many cubic yards of dirt can it move in a 39-hour week?

Anyone who wishes to become skilled in the use of the slide rule must work many problems. Those given in this book are a beginning and can serve to acquaint you with the use of the rule. However, if you continue to use the slide rule and obtain more and more practice by solving more and more problems, you will develop speed and accuracy to such an extent that the rule will become a valuable tool for use in your work. A real mastery of the slide rule is one of the most valuable things that a technician or engineer can possess.

The review problems at the end of each chapter should be worked carefully. If any difficulty arises in doing them, you may have to read part of the chapter again.

BASIS OF THE PROCESS OF MULTIPLICATION. This section is for the benefit of two classes of readers:

1. Those who must do unusual or advanced problems.

2. Those interested because of a liking for mathematics. It is possible to operate the slide rule efficiently without knowing or understanding the basis upon which this operation depends.

Slide rule operations are based on logarithms. The logarithm of a number to the base 10 is the power to which 10 must be raised to equal the number. Thus the logarithm of 100 is 2 since 10 must be raised to the power 2 to equal 100, the logarithm of 1000 is 3 since 10 must be raised to the power 3 to equal 1000, etc. The logarithm of *257* is *2.4099* since 10 must be raised to the power *2.4099* to equal *257*.

The part of a logarithm to the left of the decimal point is called the *characteristic*. In general the characteristic is one less than the digit count for the number. Thus, the characteristic of the logarithm of *257* is *2* since the digit count of 257 is 3.

The part of the logarithm to the right of the decimal point is called the *mantissa*. The mantissa of the logarithm of *257* is *4099*. The mantissa is ordinarily obtained from a table of logarithms but can often be found more efficiently with the slide rule. (See the discussion of "The Log Scale.") No matter where the decimal point of a number is located, the mantissa of its logarithm to the base 10 is always the same. Thus the mantissa of the sequence of numbers *257* is *4099* whether the number is *0.000257; 25700;* etc.

Multiplication with the slide rule is based on the fact that the logarithm of the product of two numbers is equal to the sum of the logarithms of the two numbers. An example, the multiplication of *107* by *15.3,* will illustrate this. Using logarithms to the base 10,

$$\begin{array}{ll} \text{Logarithm of } 107 & = 2.0294 \\ \text{Logarithm of } 15.3 & = \underline{1.1847} \\ \text{Sum } \ldots\ldots\ldots\ldots & = 3.2141 \end{array}$$

The sum of the logarithms of *107* and *15.3* is *3.2141*. Hence, *3.2141* is the logarithm of the product of *107* and *15.3*. The mantissa, *2141,* gives the sequence of numbers in the answer as *16374* (from a table of logarithms*). The characteristic of the logarithm of the answer is *3* so the digit count for the answer is 4. Thus the

*These figures can of course be found by use of the log scale of the slide rule; however only three digits can be read accurately.

product of *107* and *15.3* is *1637.4.* It is to be noted that the mantissa, *2141,* of the logarithm of the answer, determined the sequence of numbers in the answer. The characteristic, *3,* served only to locate the decimal point.

This example would be done on the slide rule by using the C and D scales. The settings of the slide and runner are shown in Fig. 23. The steps in the operation are:

1) Place the left index of the C scale on *107* on the D scale.
2) Set the hairline of the runner to *15.3* on the C scale.
3) Read the answer, *1637,* on the D scale under the hairline.

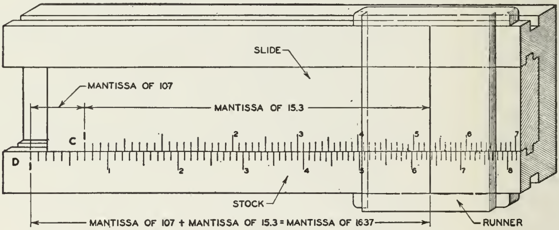


Fig. 23

Since the distance from the left end of each scale to a particular number represents, not the number, but the mantissa of the logarithm of the number, this operation actually adds the mantissa of the logarithm of the multiplicand, *107,* to the mantissa of the logarithm of the multiplier, *15.3,* to give the mantissa of the logarithm of the answer, *1637.* The mantissa for a number does not depend upon the location of the decimal point of the number, so the position of the hairline on the D scale gives only the sequence of numbers in the answer and does not locate the decimal point. The location of the decimal point can usually be estimated satisfactorily, but the operator need not rely upon this; a precise rule can be formulated, based on characteristics. If the mantissa is regarded as a decimal fraction, the addition of two mantissae would give either a number less than one or a number between one and two. If less than one, say *.2141,* the characteristics of the two num-

bers to be multiplied would add without any "carry-over" from the mantissae, so the characteristic of the answer would be just the sum of the characteristics of the two numbers of the operation. For example:

$$\begin{array}{ll} \text{Logarithm of } 128 & = 2.1072 \\ \text{Logarithm of } 35.6 & = \underline{1.5514} \\ \text{Sum} \ldots\ldots\ldots\ldots & = 3.6586 \end{array}$$

This rule applies to all cases in which the slide projects to the right of the stock and can be put into a more convenient form as an equation:

Characteristic of multiplicand + characteristic of multiplier = characteristic of answer

The characteristic is one less than the digit count for a number; that is,

Digit count for multiplicand − 1 = characteristic of multiplicand
Digit count for multiplier − 1 = characteristic of multiplier
Digit count for answer − 1 = characteristic of answer

Substituting these in the foregoing equation gives:

(digit count for multiplicand − 1) + (digit count for multiplier − 1) = (digit count for answer − 1)

Canceling a − *1* on each side,

(digit count for multiplicand + digit count for multiplier) − 1 = digit count for answer

Thus, in all multiplications in which the slide extends to the right of the stock, the digit count for the answer is one less than the sum of the digit count for the multiplicand and the digit count for the multiplier.

If the sum of the mantissae of the multiplicand and multiplier is greater than one, say *1.6893,* there is a "carry-over" of one from the mantissae to the characteristics, so the characteristic of the answer is one more than the sum of the characteristics of the two numbers of the operation. For example:

$$\begin{array}{ll} \text{Logarithm of } 8.33 & = 1.9206 \\ \text{Logarithm of } 0.642 & = \underline{0.8075} \\ \text{Sum} \ldots\ldots\ldots\ldots & = 2.7281 \end{array}$$

This applies to all cases in which the slide extends to the left of the stock. As an equation:

Characteristic of multiplicand + characteristic of multiplier + 1 = characteristic of answer

Putting this in terms of digit count, and remembering that the characteristic of a number is one less than its digit count,

(digit count for multiplicand − 1) + (digit count for multiplier − 1) + 1 = (digit count for answer − 1).

The − 1's and the + 1 cancel, so,

digit count for multiplicand + digit count for multiplier = digit count for answer

Thus, in any multiplication in which the slide extends to the left of the stock, the digit count for the answer is equal to the sum of the digit count for the multiplicand and the digit count for the multiplier.

Use of the rules for locating the decimal point is illustrated in the discussion on location of the decimal point.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown at the back of the book.

1. $37.3 \times 0.612 = ?$
2. $9.19 \times 4.72 = ?$
3. $1.032 \times 0.566 = ?$
4. $0.273 \times 3.14 = ?$
5. $328 \times 256 = ?$
6. $0.00637 \times 29.7 = ?$
7. $15.7 \times 1.83 = ?$
8. $0.755 \times 96 = ?$
9. $11.54 \times 13{,}000 = ?$
10. $2.16 \times 8.92 \times 0.495 = ?$
11. $385 \times 0.06 \times 3.07 = ?$
12. $7.77 \times 1.02 \times 128 = ?$
13. $525 \times 0.071 \times 0.95 = ?$
14. $99 \times 88 \times 77 = ?$
15. $0.344 \times 4.83 \times 0.61 \times 20.1 = ?$

16. Gravel weighs 120 pounds per cubic foot. There are 27 cubic feet in one cubic yard. How much does a cubic yard of gravel weigh?

17. A certain automobile can travel 18.3 miles on a gallon of gasoline. How far can it go on 7.5 gallons?

18. On a certain streetcar line, the average number of passengers per trip is 96. How many are carried in 63 trips?

19. If 26 pounds of paper are purchased at 85 cents per pound, what is the total cost?

20. Calculate 93 per cent of each of the following numbers: 1150; 1670; 527; 393; 67.5; 254.

21. One gallon of paint will cover 450 square feet of surface. How many square feet will 25 gallons cover?

22. A certain tank truck has a capacity of 8700 gallons. How many gallons in 17 truckloads?

23. The pressure due to a one-foot depth of water is 0.433 pounds per square inch. What is the pressure due to a depth of 34 feet?

24. $92 \times 56 = ?$
25. $75.5 \times 0.123 = ?$
26. $2.95 \times 4.78 = ?$
27. $0.307 \times 69.5 = ?$
28. $0.0587 \times 1.765 = ?$
29. $100.6 \times 0.626 = ?$
30. $8.33 \times 25.6 = ?$
31. $52.5 \times 0.444 = ?$
32. $23.7 \times 4.15 \times 0.642 = ?$
33. $1.87 \times 3.93 \times 77.3 = ?$
34. $2.02 \times 534 \times 0.065 = ?$
35. $78 \times 89 \times 0.00235 = ?$
36. $29{,}600{,}000 \times 0.000471 \times 1.25 = ?$
37. $16{,}700 \times 0.87 \times 2.4 = ?$
38. $0.143 \times 7.02 \times 55.5 = ?$
39. $9.75 \times 0.94 \times 0.89 \times 78.5 = ?$
40. $27.6 \times 5280 \times 0.169 \times 3.7 = ?$

OTHER TYPES OF SLIDE RULES. All slide rules have C and D scales located on the front. Multiplication should be done in the same way, no matter what type of slide rule you use. The proper method has been explained thoroughly in the foregoing pages of this chapter.

REVIEW PROBLEMS

You should use these problems to check your knowledge of this chapter. If you cannot solve them, you must study the chapter more carefully. Answers to Review Problems are not given in the back of the book. Readers who are working alone may check their answers by working the problems in reverse order or by doing the problems longhand.

1. How do you adjust the slide rule in order to locate the multiplicand?
2. On which scale is the answer read?
3. What is the product of 2.5×3.14?
4. Multiply $0.866 \times 0.707 \times 80$.
5. On which scale is the multiplier located?
6. $69.5 \times 1256 = ?$
7. $18000 \times 32.4 = ?$
8. $10.08 \times 99 = ?$
9. $1.23 \times 1.234 = ?$
10. $49.8 \times 20.2 = ?$
11. $14.3 \times 7 = ?$

12. $37.5 \times 2.67 = ?$

13. $528 \times 3070 = ?$

14. $0.00021 \times 939 = ?$

15. $25 \times 0.0067 = ?$

16. $3.14 \times 4.4 \times 2.86 = ?$

17. $0.043 \times 842 \times 0.072 = ?$

18. $2.06 \times 34.6 \times 18.5 = ?$

19. $0.785 \times 570 \times 4.8 \times 45.2 = ?$

20. $8.01 \times 0.693 \times 93 \times 0.0101 = ?$

21. An automobile driver maintains an average speed of 38 miles per hour. How far does he go in 8.5 hours?

22. A bagging machine can bag 23 bags of coffee in one minute. How many can it bag in 9 hours?

23. Multiply each of the following numbers by 0.866: 20; 28.7; 323; 41.4; 0.560; 7.85; 845.

24. Water weighs 62.4 pounds per cubic foot. Calculate the weight of each of the following volumes of water. Each is given in cubic feet. 0.333; 488; 51.5; 0.0624; 6.65; 9150.

25. A certain type of machine screw costs $0.00162 to make. Calculate the cost, to the nearest cent, of each of the following numbers of screws: 12; 144; 1728; 192; 48; 72; 84; 96.

26. Which number do you set with the hairline of the runner, the multiplicand or the multiplier?

27. State the rules for locating the decimal point in the product of two numbers.

28. A certain shaper in a machine shop makes 58 cutting strokes per minute. How many strokes does it make in one hour?

29. One gallon of water weighs 8.35 pounds. How much will 26 gallons weigh?

30. Steel plate one-fourth of an inch thick weighs 10.2 pounds per square foot. How much does a square piece of plate, 6.2 feet by 5.3 feet, weigh?

31. An assembly line produces 840 units per day. How many units will it produce in 46 days?

32. A testing machine operator can test 11 steel specimens in one hour. How many can he test in 6 seven-hour days?

33. A certain type of steel spring costs 3.6 cents to manufacture. Calculate the cost of 12 dozen springs to the nearest cent.

34. A steel cable, 382 ft. long, is stretched 0.000537 inch per inch. What is the total stretch?

35. Along a riveted joint, 29 rivets are to be equally spaced. The pitch (spacing center to center) is to be 3.21 inches. Find the distance from the center of the rivet on one end to the center of the rivet on the other end.

CHAPTER V

DIVISION

THE PROCESS OF DIVISION. Two numbers are involved in the ordinary process of division. The first number is the *dividend,* the number which is divided. The second number is the *divisor,* the number which divides into the dividend. For example, in dividing *39* by *7, 39* is the dividend and *7* is the divisor. This operation can also be symbolized as $39 \div 7$ or $\frac{39}{7}$. The answer is called the *quotient.*

STEPS IN THE PROCESS OF DIVISION

1) In dividing one number by another, the hairline of the runner is set to coincide with the first number, or dividend, on the D scale.
2) Then the slide is moved to such a position that the second number, or divisor, on the C scale is under the hairline.
3) In this setting only one index of the C scale can be between the ends of the D scale. The answer, or quotient, is read on the D scale, under this index of the C scale.

> **Rule 2. Division.** *Set the hairline of the runner to the dividend on the D scale. Then slide the divisor on the C scale under the hairline. Finally, read the answer on the D scale under one index of the C scale.*

ILLUSTRATIVE EXAMPLES

1. To divide 167 by 1.16. Fig. 24 shows the proper setting of the slide and runner for this problem. It should be done in the following steps:

1) Set the hairline of the runner to *167,* the dividend, on the D scale.
2) Move the slide so that *1.16,* the divisor, on the C scale is under the hairline.
3) The left index of the C scale remains between the ends of the D scale. Read the answer, *144,* on the D scale under the left index of the C scale.

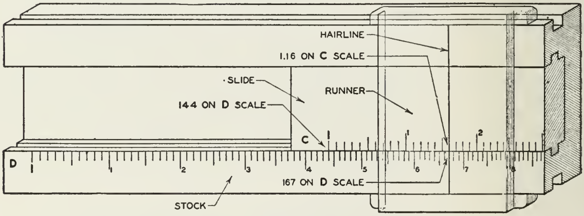


Fig. 24

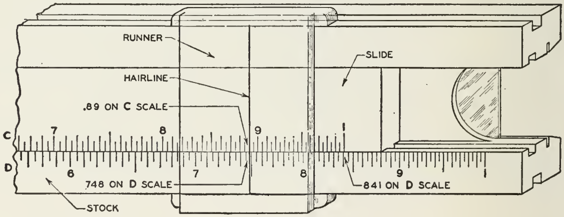


Fig. 25

2. To divide 748 by 0.89. The correct setting of the slide and runner is shown in Fig. 25. The steps in the problem are:

1) Set the hairline of the runner to the dividend, *748,* on the D scale.
2) Place the slide so that the divisor, *0.89,* on the C scale is under the hairline.
3) Read the answer, *841,* on the D scale under the right index of the C scale.

3. To divide 632 by 1.43. Fig. 26 shows how the slide and runner should be set for this problem. The procedure is:

1) Locate the dividend, *632,* on the D scale by means of the hairline on the runner.
2) Adjust the slide so that the divisor, *1.43,* on the C scale is under the hairline.
3) Read the answer, *442,* on the D scale under the left index of the C scale.

There is never a question as to which index of the C scale locates the answer. Only one index can give a reading on the D scale at one time and this is the proper index to use.

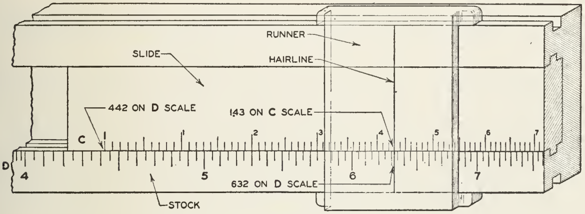


Fig. 26

PRACTICE PROBLEMS

These problems are furnished for you to practice division. What you have just read is very simple, but you will forget it quickly unless you do many problems. Work only for the sequence of numbers, ignoring the decimal.

After you have worked all of the following problems, compare your answers with the correct answers shown at the back of the book.

1. $390 \div 0.7 = ?$

2. $299 \div 1.73 = ?$

3. $69.3 \div 0.08 = ?$

4. $283 \div 2.08 = ?$

5. $97 \div 0.13 = ?$

6. An engine runs at 4320 revolutions per minute. How many revolutions is this per second?

7. A certain type of steel drill is listed at $8.33 per dozen. Calculate the price of one drill to the nearest cent.

8. Divide 17,000 by 47.

9. A core molder, working at piece rates, earned $30.10 in a 35-hour week. In cents, what was his average hourly earning?

10. Assume that 655 is 82 per cent of a number x. Find x. (*Hint:* Divide 655 by 0.82.)

11. A small plot of ground, with 16.5-foot frontage, sells for $528. How much is this per front foot?

12. A bar of cast iron, with cross-sectional area of 1.62 square inches is subjected to a load of 1,100 pounds. How much is this in pounds per square inch?

13. The total load on a steel cable is to be 86,000 pounds. The cable is to be made of steel wires, each of which can carry 112 pounds. How many wires are required for the cable?

14. The circumference of a given circle is 735 inches. What is the diameter? (*Hint:* The circumference is 3.14 times the diameter.)

15. A rectangular room is 16.5 feet long and has an area of 198 square feet. Find the width of the room.

LOCATION OF THE DECIMAL POINT IN DIVISION. Two positive rules can be stated for locating the decimal point in the answer when dividing one number by another.

1. Division. *When the slide extends to the right of the stock, the digit count* for the answer is one more than the digit count for the dividend minus the digit count for the divisor.*†

When the slide projects to the right of the stock, the answer, or quotient, is read on the D scale under the left index of the C scale. Remember that, in a number which is smaller than one, any zeros immediately following the decimal point are counted as negative digits. For example, in the number *0.00263* there are two negative digits, and the digit count is -2. The following examples will demonstrate the method of locating the decimal point in dividing one number by another.

ILLUSTRATIVE EXAMPLES

4. To divide 6200 by 150.

1) The sequence of numbers in the answer is read as *413*.

*Digit counts are explained in the chapter on Multiplication, p. 50.

†There is one exception to this rule. When the dividend is *1, 10, 100, 1,000,* etc. the digit count for the answer is always equal to the digit count for the dividend minus the digit count for the divisor, no matter which way the slide extends. The slide could go either way because you could use either the left or right index of the D scale for the dividend in such a case.

2) The digit count for the dividend, *6200,* is four; the digit count for *150* is three. Four minus three is one.
3) The slide extends to the right of the stock so add one to this difference. The result is two. Hence, the digit count for the answer is two, and it is 41.3.

5. To divide 0.00428 by 0.026.

1) The sequence of numbers in the answer is read as *1647.*
2) The digit count is two negative digits (-2) for the dividend, *0.00428,* and one negative digit (-1) for the divisor, *0.026.* Minus one subtracted from minus two is minus one,* $-2-(-1)=-1$.
3) The slide extends to the right of the stock in this operation. Therefore, you should add one to minus one. This gives zero as the digit count for the answer, which means that there are zero digits to the left of the decimal point in the answer and it is 0.1647.

6. To divide 746 by 0.000535.

1) The sequence of numbers in the answer is read as *1395.*
2) The digit count is three for the dividend, *746,* and minus three (-3) for the divisor, *0.000535.* Minus three subtracted from three is six,* $3-(-3)=6$.
3) Add one to six since the slide extended to the right of the stock. This gives a digit count of seven for the answer. Hence, there are seven digits to the left of the decimal point in the answer and it is 1,395,000.

7. To divide 0.864 by 1268.

1) The sequence of numbers in the answer is read as *682.*
2) The digit count is zero for the dividend, *0.864,* and four for the divisor, *1268.* Zero minus four is minus four, $0-4=-4$.
3) Since the slide extended to the right beyond the stock, you add one to minus four, $-4+1=-3$, the digit count for the answer. Hence there are three zeros to the right of the decimal point in the answer, and it is 0.000682.

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*For an explanation of negative numbers see page 240.

PRACTICE PROBLEMS

After you have worked all the following problems, check your answers with the correct answers at the back of the book.

1. $1.21 \div 1.006 = ?$
2. $0.932 \div 4.77 = ?$
3. $0.00817 \div 0.0000123 = ?$
4. $70.7 \div 388 = ?$
5. $3{,}160{,}000 \div 201 = ?$
6. $648 \div 0.0509 = ?$
7. $54.1 \div 0.353 = ?$
8. $0.427 \div 32.3 = ?$
9. $0.0067 \div 0.0586 = ?$
10. $9.8 \div 0.866 = ?$

2. Division. *When the slide extends to the left of the stock, the digit count for the answer is equal to the digit count for the dividend minus the digit count for the divisor.* In such a case, the answer is read on the D scale under the right index of the C scale. As before, in a number which is less than one, the zeros immediately following the decimal point are counted as negative digits.

ILLUSTRATIVE EXAMPLES

8. To divide 28.3 by 4.61.

1) The sequence of numbers in the answer is read as *614*.
2) The digit count is two for the dividend, *28.3,* and one for the divisor, *4.61*. Two minus one is one: $2 - 1 = 1$.
3) The slide extends to the left of the stock during the operation; therefore, nothing is added to this. Hence, the digit count for the answer is one; which means there is one digit to the left of the decimal point in the answer, and it is 6.14.

9. To divide 0.00143 by 37.5.

1) The sequence of numbers in the answer is read as *381.*
2) The digit count for the dividend, *0.00143,* is minus two; for the divisor, *37.5,* the digit count is plus two. Two subtracted from minus two is minus four: $-2-(+2)=-4$.
3) Since the slide extended to the left of the stock in this operation, nothing is to be added to minus four. Hence, the digit count for the answer is -4, there are four zeros to the right of the decimal point, and the answer is 0.0000381.

10. To divide 0.217 by 0.000624.

1) The sequence of numbers in the answer is read as *348.*

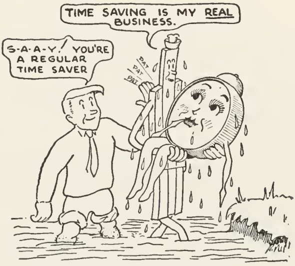
2) The digit count for the dividend, *0.217,* is zero, and minus three is the digit count for the divisor, *0.000624.* Minus three subtracted from zero is three, thus: $0 - (-3) = 3$.

3) The slide extended to the left of the stock. Therefore, nothing is added to three. Thus the digit count for the answer is three, and it is 348.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers at the back of the book.

1. $1.008 \div 256 = ?$
2. $878 \div 95.3 = ?$
3. $0.560 \div 0.667 = ?$
4. $0.000417 \div 0.00745 = ?$
5. $0.0362 \div 57.5 = ?$
6. $1728 \div 0.0977 = ?$
7. $0.0722 \div 8.42 = ?$
8. $60{,}000 \div 815 = ?$
9. $0.0109 \div 99 = ?$
10. $27.5 \div 0.0631 = ?$



THE SLIDE RULE SAVES TIME

DIVISION OF ONE NUMBER BY EACH OF MANY NUMBERS. Many problems arise in which it is necessary to divide one number by each of many others. In this process the first number is always the dividend while each of the others is in turn the divisor. The dividend can be located on the D scale by means of the hairline on the runner. Then each divisor is, in its turn, brought under the hairline by moving the C scale. For each the answer is read under that index of the C scale which remains between the ends of the D scale. The dividend need be located only once.

ILLUSTRATIVE EXAMPLE

11. To divide 89 by each of the following numbers: 93.5; 82.1; 68.5; 51.7; 44.8; 37.3; 23.5; 17. The procedure is:

1) Set hairline of runner to the dividend, *89,* on the D scale.

2) Move the slide so that the first divisor, *93.5,* on the C scale is under the hairline. Read the answer for this part of the problem on the D scale under the right index of the C scale as *0.953.*
3) Leave the runner in its position, while moving the slide so that the second divisor, *82.1,* on the C scale is under the hairline. Read the second part of the answer under the left index of the C scale as *1.082.*
4) Use the remainder of the divisors in a similar manner. The answers, respectively, are: 1.297; 1.721; 1.986; 2.39; 3.78; 5.23.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

1. Express each of the following common fractions as a decimal fraction:

$$\frac{7}{11}; \frac{7}{13}; \frac{7}{15}; \frac{7}{17}; \frac{7}{19}$$

2. Convert each of the following common fractions to a decimal fraction:

$$\frac{1}{7}; \frac{1}{13}; \frac{1}{15}; \frac{1}{21}; \frac{1}{31}$$

3. Divide 68400 by each of the following numbers: 1875; 2520; 3125; 4420; 6010.

DIVISION OF EACH OF MANY NUMBERS BY A SINGLE NUMBER. In calculating percentages, compiling tables, converting common fractions to decimal fractions, etc., it is often necessary to divide each of several numbers by a single number. The dividend changes for each part of the problem, but the divisor is always the same. Each part of the problem can be symbolized as $\frac{dividend}{divisor}$. But this can also be written as $\frac{1}{divisor} \times dividend$. (1) The value of $\frac{1}{divisor}$ can be obtained by dividing *1* by the *divisor*. Either index of the D scale can be used as the *1* in this case, and the value of $\frac{1}{divisor}$ is located on the D scale under

the proper index of the C scale. Then if $\frac{1}{divisor}$ is regarded as a multiplicand and the dividend as a multiplier, $\frac{1}{divisor}$ is located properly to be multiplied by the dividend. This is done by (2) setting the hairline of the runner to the dividend on the C scale. Then the answer is read on the D scale under the hairline. Each part of the problem is worked in the same way. Once the slide is set to the value of $\frac{1}{divisor}$, several numbers can be used in turn as dividends without moving the slide.

ILLUSTRATIVE EXAMPLES

12. Divide each of the following numbers by 972: 112; 195.3; 220; 394; 487.

1) Obtain the value of $\frac{1}{972}$. Do this by using the right index of the D scale as *1;* that is, place *972* on the C scale over the right index of the D scale. The value of $\frac{1}{972}$ is then located on the D scale under the left index of the C scale. However, you need not trouble to write down the value of $\frac{1}{972}$.

2) Multiply $\frac{1}{972}$ by each of the other numbers. This requires setting the hairline of the runner to each of the dividends in turn on the C scale. Read the answer for each number in its turn on the D scale under the hairline. The answers, respectively, are: 0.1152; 0.201; 0.227; 0.406; 0.502.

13. Convert each of the following common fractions to a decimal fraction: $\frac{11}{108}$; $\frac{15}{108}$; $\frac{24}{108}$; $\frac{29}{108}$; $\frac{37}{108}$. This requires that each numerator be divided by *108*.

1) Divide *1* by *108* to get $\frac{1}{108}$. Use the left index of the D scale

as *1* and bring *108* on the C scale over it. The value of $\frac{1}{108}$ is located on the D scale under the right index of the C scale.

2) Multiply each of the above numerators by $\frac{1}{108}$. Locate each numerator in turn on the C scale by setting the hairline of the runner to it. When a particular numerator is set, read the answer for it on the D scale under the hairline. The answers, respectively, are: 0.1018; 0.1389; 0.222; 0.269; 0.343.

You observe that the right index of the D scale was taken as *1* to divide *1* by *972* in Example 12. The left index of the D scale was taken as *1* to divide *1* by *108* in Example 13. Of course, each index of the D scale can be read as *1,* so each division could be made with either index and the result would be the same. However, when you are ready to multiply such a quotient, $\frac{1}{972}$ or $\frac{1}{108}$, etc., by the dividends of the problem, the C scale must be in such a position that you can locate the multipliers on it and read answers on the D scale. An example will illustrate the point.

ILLUSTRATIVE EXAMPLE

14. Divide 443 and 518 by 71.

1) Obtain $\frac{1}{71}$ by using the left index of the D scale as *1*. Divide it by *71*.
2) Try to multiply by *443* or *518*. You cannot do this since the part of the C scale you need to use is to the left of the stock.
3) Now obtain $\frac{1}{71}$ by using the right index of the D scale as *1*. Divide it by *71*.
4) Multiply this by *443* and *518*. The answers are 6.24 and 7.30.

You can always multiply $\frac{1}{divisor}$ by any *dividend*. However, you must use the proper index of the D scale as *1* in obtaining $\frac{1}{divisor}$. A good rule is: If the sequence of numbers (without

regard for the decimal point) in the dividend is smaller than the sequence of numbers (without regard for the decimal point) in the divisor, use the right index of the D scale; if larger, use the left index of the D scale. In some problems, as in the following example, you may have to use the right index of the D scale for part of the calculations and the left index of the D scale for the remainder.

ILLUSTRATIVE EXAMPLE

15. Divide each of the following numbers by 5.23: 1.18; 18.52; 322; 59.8; 716; 9.27.

1) Obtain $\frac{1}{5.23}$ by using the right index of the D scale as *1*. Divide it by *5.23*.
2) Multiply this result by *1.18; 18.52;* and *322* in turn. The answers are 0.226; 3.54; and 61.6, respectively.
3) Obtain $\frac{1}{5.23}$ by using the left index of the D scale as *1*. Divide it by *5.23*.
4) Multiply by *59.8; 716;* and *9.27* in turn. The answers are 11.42; 137; and 1.772, respectively.

PERCENTAGE CALCULATIONS. A very common type of calculation is that in which it is required to find what per cent a given number x is of another number, y. To do this, you divide x by y and multiply the result by *100*. Multiplication by *100* does not require any manipulation of the slide rule. You simply write down the value of $\frac{x}{y}$ and set the decimal point two places to the right of the normal location. Then you have the per cent which x is of y.

ILLUSTRATIVE EXAMPLE

16. What per cent is 68 of 94?

1) Divide *68* by *94*. The result is *0.723*.
2) To change this to per cent, multiply by *100*. This only requires that you move the decimal point two places to the right. The answer is 72.3 per cent.

Many of the problems in which it is necessary to divide each of many numbers by a single number are percentage calculations.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown at the back of the book.

1. Convert each of the following common fractions to a decimal fraction:

$$\frac{11}{66}; \frac{17}{66}; \frac{23}{66}; \frac{31}{66}; \frac{47}{66}; \frac{57}{66}; \frac{65}{66}$$

2. The numbers 202; 287; 413; 562; and 833, represent volumes in cubic inches. Convert each to cubic feet. (*Hint:* There are 1728 cubic inches in one cubic foot.)

3. The cost of one dozen of each of a number of articles is, respectively, \$1.90; \$3.50; \$6.80; \$8.30; \$9.20; and \$11.60. Calculate the cost of a single article of each kind to the nearest cent.

4. Find what per cent each of the following numbers is of 616: 12; 208; 317; 493; 654; 76.5; 8.43.

5. A speed of 1 mile per hour is equivalent to 1.467 feet per second. Convert each of the following speeds in feet per second to miles per hour: 20; 28; 39; 46; 63; 88; and 110.

6. Each of the following numbers is 63 per cent of a certain other number: 101.5; 234; 501; 67.3; 786; 8.27. Find the other number for each. (*Hint:* Divide each number by 0.63.)

7. Divide each of the following numbers by 86.3: 2.77; 946; 62.4; 9.03; 10.78; 87.5.

8. Convert each of the following common fractions to a decimal fraction:

$$\frac{12}{17}; \frac{5}{17}; \frac{7}{17}; \frac{8}{17}; \frac{14}{17}; \frac{16}{17}$$

9. Find what per cent each of the following numbers is of 42.7: 4.95; 16.06; 8.8; 26.2; 3.95; 9.48.

10. Convert each of the following common fractions to a decimal fraction:

$$\frac{3}{7}; \frac{11}{70}; \frac{47}{700}; \frac{8}{70}; \frac{90}{7{,}000}; \frac{92}{700}; \frac{63}{70}; \frac{5}{7}$$

DIVISION OF ONE NUMBER BY THE PRODUCT OF TWO NUMBERS. Many engineering and shop formulae lead to the division of one number by the product of two

others. For example, suppose you want the result of $\dfrac{7{,}200}{3.14 \times 14.8}$. It is very convenient to be able to obtain such a result by continuous manipulation of the slide rule without writing down any intermediate answer. The numbers in such a problem can be designated as the *dividend,* the *first divisor* (in the foregoing example the first divisor is *3.14*), and the *second divisor* (in this example the second divisor is *14.8*). The calculation is started by (1) setting the hairline of the runner to the dividend on the D scale. (2) The first divisor on the C scale is brought under the hairline by moving the slide. The result of this part of the calculation can be read on the D scale under the proper index of the C scale. There is no need, however, to write it down; simply (3) move the runner so that the hairline is on this index of the C scale. (4) Next adjust the slide so that the second divisor on the C scale is under the hairline, and (5) the final answer is read on the D scale under the index of the C scale that is between the ends of the D scale. In the following examples, work only for the sequence of numbers, ignoring the decimal.

ILLUSTRATIVE EXAMPLES

17. Find the result of $\dfrac{7{,}200}{3.14 \times 14.8}$.

1) Locate the dividend, *7,200,* on the D scale by means of the hairline of the runner.
2) Move the slide so that the first divisor, *3.14,* on the C scale is under the hairline.
3) Leave the slide in this position while moving the hairline of the runner to the left index of the C scale.
4) Adjust the slide so that the second divisor, *14.8,* on the C scale is under the hairline.
5) Read the answer, *155,* on the D scale under the left index of the C scale.

18. Find the result of $\dfrac{18{,}560}{550 \times 1.3}$.

1) Set the hairline of the runner to the dividend, *18,560,* on the D scale.
2) Slide the first divisor, *550,* on the C scale under the hairline.

3) Let the slide remain in its position. Move the runner so that its hairline coincides with the right index of the C scale.
4) Bring the second divisor, *1.3,* on the C scale under the hairline of the runner.
5) Read the final answer, *26,* on the D scale under the left index of the C scale.

PRACTICE PROBLEMS

In the following problems, work only for sequence of numbers, ignoring the decimal point. *After* you have worked all of them, check your answers with the correct answers shown in the back of the book.

1. $\dfrac{97}{38 \times 1.2} = ?$

2. $\dfrac{2{,}730}{0.785 \times 745} = ?$

3. $\dfrac{5.45}{1.1 \times 1.05} \simeq ?$

4. $\dfrac{1{,}728}{220 \times 0.0216} = ?$

5. $\dfrac{0.866}{0.693 \times 0.707} = ?$

LOCATION OF DECIMAL POINT. The procedure for locating the decimal point in the answer when dividing one number by the product of two others is:

1. Add the digit count for the first divisor to the digit count for the second divisor.

2. Subtract this sum from the digit count for the dividend.

3. Add one to this result for each part of the operation in which the slide extends to the right of the stock. You may find it desirable to make a mark on the paper for each time that the slide extends to the right during the problem.

ILLUSTRATIVE EXAMPLES

19. Find the result of $\dfrac{3{,}040}{18.7 \times 0.011}$.

The sequence of numbers in the answer is read as *1479.* Notice that the slide extended to the right of the stock twice, once during each part of the operation.

1) For the first divisor, *18.7,* the digit count is two; for the second divisor, *0.011,* it is minus one. The sum of two and minus one is one: $2 + (-\ 1) = 1$.

2) The digit count for the dividend, *3,040,* is four. Subtract one from four, leaving three.
3) Add two to this, since the slide extended to the right of the stock during each of the two parts of the problem. The result is five, the digit count for the answer. Hence there are five digits to the left of the decimal point in the answer and it is 14,790.

20. Calculate $\dfrac{12{,}560}{29{,}000{,}000 \times 0.785}$.

The sequence of numbers in the answer is read as *551.* Notice that the slide did not extend to the right of the stock at all during the operation.

1) The digit count is eight for the first divisor; *29,000,000;* and zero for the second divisor, *0.785.* The sum of eight and zero is eight.
2) The digit count for the dividend, *12,560,* is five. Subtract eight from five, leaving minus three: $5 - (+8) = -3$.
3) Since the slide did not extend to the right of the stock, there is nothing to add to minus three. Hence, -3 is the digit count for the answer, there are three zeros to the right of the decimal point in the answer, and it is 0.000551.

21. Find the result of $\dfrac{0.00247}{0.0000083 \times 16.7}$.

The sequence of numbers in the answer is read as *1782.* Notice that the slide extended to the right of the stock during one part of the operation.

1) The digit count is minus five for the first divisor, *0.0000083,* and plus two for the second divisor, *16.7.* The sum of minus five and plus two is minus three. $-5 + 2 = -3$.
2) The digit count for the dividend, *0.00247,* is minus two. Subtract minus three from minus two, leaving plus one:* $-2 - (-3) = -2 + 3 = +1$.
3) Add one to this, since the slide extended to the right of the stock once during the problem. The result is two. So the digit count for the answer is $+2$, there are two digits to the left of the decimal point in the answer, and it is 17.82.

*For an explanation of negative numbers, see p. 240.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown at the back of the book.

1. $\dfrac{833}{0.442 \times 5} = ?$

2. $\dfrac{0.0128}{27.7 \times 0.00176} = ?$

3. $\dfrac{4.12}{0.355 \times 9.6} = ?$

4. $\dfrac{67.2}{7.24 \times 32.2} = ?$

5. $\dfrac{5{,}280}{3{,}600 \times 7.5} = ?$

6. $\dfrac{0.244}{0.0045 \times 4{,}180} = ?$

7. $\dfrac{9{,}400}{11{,}400{,}000 \times 0.379} = ?$

8. $\dfrac{3.75}{21.5 \times 0.137} = ?$

9. $\dfrac{0.000062}{0.00047 \times 0.0263} = ?$

10. $\dfrac{296}{83.4 \times 1.020} = ?$

BASIS OF THE PROCESS OF DIVISION. This explanation is written for the benefit of those who wish a thorough knowledge of the slide rule and who may have occasion to make rather unusual types of calculations. Frequently, a complete understanding of the slide rule will make it possible to develop short cuts and individual timesavers. However, it is possible to divide one number by another without knowing the basis of the process.

Division with the slide rule is based on logarithms. It rests on the fact that the logarithm of the quotient of two numbers is equal to the logarithm of the dividend minus the logarithm of the divisor. In dividing *17.1* by *1.33,* for example, the logarithm of the quotient, or answer, is equal to the logarithm of the dividend, *17.1,* minus the logarithm of the divisor, *1.33.* Using a table of logarithms to the base 10,

$$\begin{array}{rl}
\text{Logarithm of dividend, 17.1} & = 1.2330 \\
\text{Logarithm of divisor, 1.33} & = \underline{0.1239} \\
\text{Difference} & = 1.1091
\end{array}$$

The quotient is the number which has *1.1091* for its logarithm. With the aid of the tables, the quotient is found to be *12.86.* The

setting of the slide rule for this division is shown in Fig. 27. In each case, the distance from the left end of the scale to the location of the number represents the mantissa of the logarithm to the base 10.

The mantissa of the logarithm is the part of the logarithm to the right of the decimal point. It gives the sequence of numbers and has nothing to do with the location of the decimal point of the number. The characteristic* of the logarithm, which is the part to the left of the decimal point, shows the location of the decimal point of the number. You will recall that the characteristic of the logarithm of a number is one less than the digit count for the number.

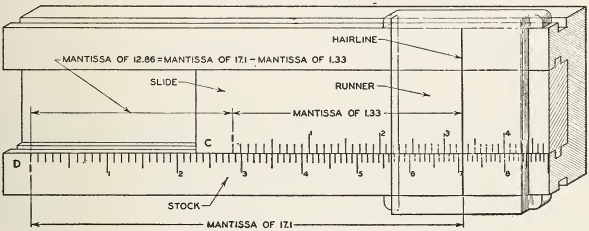


Fig. 27

Fig. 27 shows that the slide rule operation actually subtracts the mantissa of the logarithm of the divisor, *1.33,* from the mantissa of the dividend, *17.1,* to give the mantissa of the quotient, *12.86.* Thus the sequence of numbers in the answer is determined. Note that, as stated before, the mantissa gives only the sequence of numbers and does not locate the decimal point; that comes from the characteristic. If the slide of the rule extends to the right of the stock during the division, the mantissa of the divisor is less than that of the dividend. Hence, there is no "carryover" to the characteristics in subtraction, and the characteristics can be subtracted separately. The characteristic of the logarithm of the quotient is equal to the characteristic of the dividend minus the characteristic of the

*See the explanation of characteristics on p. 40.

divisor. The logarithm of the quotient in this example is 1.1091 and its characteristic is *1*. This means there are two digits to the left of the decimal point in the answer, so it is *12.86*. The decimal point can be located by use of characteristics, but it is more convenient to use the digit counts for the dividend and divisor. As an equation,

characteristic of dividend — characteristic of divisor =
characteristic of quotient

Remembering that,

characteristic of a number = digit count for the number — 1

the foregoing equation can be written,

(digit count for dividend — 1) — (digit count for divisor — 1) =
(digit count for quotient — 1)

Canceling a — 1 on each side,

(digit count for dividend — digit count for divisor) + 1 =
digit count for quotient

Hence when the slide extends to the right during the division, the digit count for the quotient is one more than the digit count for the dividend minus the digit count for the divisor.

If the slide extends to the left of the stock, the mantissa of the logarithm of the divisor is greater than the mantissa of the dividend. Hence in subtraction, there is a "carryover" to the characteristics, and the characteristic of the quotient is one less than the characteristic of the logarithm of the dividend minus the characteristic of the divisor. As an equation, this is,

characteristic of dividend — characteristic of divisor — 1 =
characteristic of quotient

When the equation is put in terms of digits, it becomes,

(digit count for dividend — 1) — (digit count for divisor — 1) —
1 = (digit count for quotient — 1)

Canceling all — 1's,

digit count for dividend — digit count for divisor =
digit count for quotient

Hence, when the slide extends to the left of the stock during the division, the digit count for the quotient is equal to the digit count for the dividend minus the digit count for the divisor.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown at the back of the book.

1. $96.6 \div 38.3 = ?$
2. $278 \div 53.7 = ?$
3. $0.472 \div 11.7 = ?$
4. $301 \div 81.2 = ?$
5. $2.17 \div 1.93 = ?$
6. $34{,}700 \div 368 = ?$
7. $5280 \div 65.5 = ?$
8. $43.4 \div 48.2 = ?$
9. $1380 \div 21.3 = ?$
10. $0.866 \div 0.0589 = ?$
11. $1.682 \div 29.1 = ?$
12. $32.2 \div 5.7 = ?$
13. $7.25 \div 1.28 = ?$
14. $18{,}000 \div 2.42 = ?$
15. $989 \div 0.0765 = ?$
16. $112 \div 0.785 = ?$
17. $25.4 \div 8.67 = ?$
18. $0.346 \div 1.53 = ?$
19. $8320 \div 63.7 = ?$
20. $536 \div 28.1 = ?$

Convert the following fractions to decimals.

21. $\frac{3}{8}; \frac{3}{13}; \frac{3}{17}$
22. $\frac{5}{9}; \frac{5}{11}; \frac{5}{12}$
23. $\frac{4}{21}; \frac{5}{21}; \frac{11}{21}$
24. $\frac{11}{87}; \frac{24}{87}; \frac{31}{87}$
25. $\frac{14}{109}; \frac{19}{109}; \frac{23}{109}$

Work the following:

26. $\dfrac{64.7}{21 \times 0.317} = ?$
27. $\dfrac{135.7}{5.4 \times 9.2} = ?$
28. $\dfrac{0.195}{2.31 \times 0.415} = ?$
29. $\dfrac{15{,}600}{28.2 \times 0.813} = ?$
30. $\dfrac{888}{75.2 \times 1.62} = ?$

OTHER TYPES OF SLIDE RULES. No matter what type of slide rule you use, problems of division should be worked by the method described in this chapter. All slide rules have the C and D scales on the lower part of the front.

REVIEW PROBLEMS

Answers to Review Problems are not given in the back of the book. Readers who are working alone may check their answers by working the problems longhand.

1. A rectangular vat has floor dimensions of 7.5 feet and 6.2 feet. Its volume is 113 cubic feet. Find its height.

2. Convert $\frac{5}{12}$ to a decimal fraction.

3. A motorist travels 332 miles in 8.25 hours. What is his average speed?

4. Eighteen rivets are to be equally spaced in a row so that the distance from the center of the rivet on one end to the center of the rivet on the other end is 41.8 inches. What should be the distance between centers of adjacent rivets?

5. Steel plate 1 inch thick weighs 40.8 pounds per square foot. How thick is plate which weighs 7.65 pounds per square foot?

6. On what scale is the dividend located?

7. A mason can lay 820 bricks in an eight-hour day. What is the average number laid per minute?

8. There are 43,560 square feet in one acre. How many acres are there in 273,000 square feet?

9. Convert each of the following common fractions to a decimal fraction:

$$\frac{3}{128};\ \frac{7}{128};\ \frac{11}{128};\ \frac{19}{128};\ \frac{34}{128};\ \frac{63}{128};\ \frac{105}{128}$$

10. Find what per cent 117 is of 483.

11. What is $\frac{1}{24}$ of $\frac{1}{32}$?

12. Solve the following equation for x:

$$0.00824\,x = 153$$

13. A manufacturer pays $13,440 for 39,500 pounds of copper. What is the cost per pound?

14. On what scale is the divisor located?

15. The circumference of a certain circle is 484 feet. What is its diameter?

16. A certain pump has a rate of 96 gallons per minute. How long must it operate to fill a tank which has a capacity of 6,900 gallons?

17. Calculate $\frac{11}{29}$ of $\frac{1}{3}$.

18. A trucking company's bill for 11,500 gallons of gasoline was $2,060. What was the cost per gallon?

19. Each of the following numbers is 28.5 per cent of another number: 1.38; 21.2; 3,430; 7.36; 0.991. Calculate each of the other numbers.

20. A manufacturer plans to build a storage bin with a capacity of 1,750 cubic feet. He has available space 19.7 feet wide and building regulations prohibit storing the particular material to a depth greater than 6 feet. How long a space does he need?

21. $\dfrac{7{,}300}{2{,}550{,}000 \times 4.3} = ?$

22. $\dfrac{16{,}550}{0.000573} = ?$

23. $\dfrac{97.5}{1{,}030} = ?$

24. $\dfrac{1.05}{0.932} = ?$

25. $\dfrac{30.4}{30.6} = ?$

26. $\dfrac{50{,}500}{70.7} = ?$

27. $\dfrac{895}{0.866 \times 0.693} = ?$

28. $\dfrac{0.299}{37.8 \times 0.00061} = ?$

29. $\dfrac{43}{3.14 \times 6 \times 9.3} = ?$

30. $\dfrac{0.000632}{0.0000123} = ?$

CHAPTER VI

COMBINATIONS OF MULTIPLICATION AND DIVISION

Most problems in science and technology can be solved by the use of formulae, formulae which can be found in handbooks, reference books and textbooks. The usual procedure is to find the proper formula and substitute in it those numbers which fit in the particular example. Then the answer is obtained by carrying out the calculations. Many formulae lead to calculations involving several operations of multiplication and division, for example,

$$\frac{5 \times 4750 \times 13{,}820{,}000}{384 \times 30{,}000{,}000 \times 133.5}$$

It is in this type of calculation that the slide rule appears to greatest advantage, since the calculation can be carried all the way through to the final answer without writing down any intermediate results. The savings in time and effort are enormous. Also, it is so much simpler to do it with the slide rule that you are less likely to make an error than if you do it longhand. General rules can be stated for performing such a calculation, but are apt to become complicated and long. For this reason, the procedure will be described by the use of examples. Follow them carefully with your own slide rule.

SLIDE RULE STUDY REQUIRES CONCENTRATION

ILLUSTRATIVE EXAMPLES

1. Find the result of $\dfrac{5 \times 4750 \times 13{,}820{,}000}{384 \times 30{,}000{,}000 \times 133.5}$

In such a problem it is best to work from left to right and to perform alternately the processes of division and multiplication. The procedure is:

1) Set the hairline of the runner to the first number in the numerator, in this case *5,* on the D scale.
2) Divide this by the first number in the denominator, *384,* by bringing *384* on the C scale under the hairline.
3) Hold the result of this operation on the D scale under the left index of the C scale, and multiply by the second number in the numerator, *4750,* by setting the hairline of the runner to *4750* on the C scale.
4) The result of this much of the problem is on the D scale under the hairline. Leave the runner where it is and divide by the second number of the denominator, *30,000,000* by bringing *30,000,000* on the C scale under the hairline.
5) Leave the slide fixed and multiply by the third number in the numerator, *13,820,000,* by setting the hairline of the runner to *13,820,000* on the C scale.
6) Let the runner stay where it is and divide by the third number in the denominator, *133.5,* by bringing *133.5* on the C scale under the hairline.
7) Read the final answer, *0.214,* on the D scale under the left index of the C scale.

Do not worry, at this time, about locating the decimal point in the answer. That comes later. Ignore it just now and concentrate on getting only the sequence of numbers in the answer. Be sure that no number is left out and that no number is used twice.

In Example 1 there are six separate numbers to be set on the slide rule. It is reasonable to expect a slight error in setting each one. These errors may accumulate so that your answer will differ from the one given here in the third digit. However, any such error should be less than one per cent. Experience in setting numbers will give increased precision and errors will be reduced.

2. Find the result of $\dfrac{225 \times 52 \times 32}{37 \times 29}$

1) Set the hairline of the runner on the first number of the numerator, *225,* on the D scale.

2) Divide by the first number in the denominator, *37,* by bringing *37* on the C scale under the hairline of the runner.
3) Multiply by the second number in the numerator, *52,* by setting the hairline of the runner to *52* on the C scale.
4) Divide by the second number of the denominator, *29,* by bringing *29* on the C scale under the hairline of the runner.
5) Multiply by the third number of the numerator, *32,* by setting the hairline of the runner to *32* on the C scale.
6) Read the answer, *349,* on the D scale under the hairline.

A problem of this sort can be done in much less time than is required to tell how to do it. This one, for example, can be worked in less than ten seconds by anyone who is familiar with the slide rule, and no strenuous mental activity is involved.

3. Calculate $\dfrac{98 \times 144 \times 1.25}{4 \times 0.375}$

1) Set the hairline of the runner to the first number of the numerator, *98,* on the D scale.
2) Divide by the first number of the denominator, *4,* by moving *4* on the C scale under the hairline of the runner.
3) Multiply by the second number of the numerator, *144,* by setting the hairline of the runner to *144* on the C scale.
4) Divide by the second number of the denominator, *0.375,* by bringing *0.375* on the C scale under the hairline of the runner.
5) Multiply by the third number of the numerator, *1.25,* by setting the hairline of the runner to *1.25* on the C scale.
6) Read the answer, *11,750,* on the D scale under the hairline of the runner.

Notice that each number of the numerator is located by moving the hairline of the runner while each number of the denominator is located on the C scale under the hairline by moving the slide.

It is time now to mention another important matter, that of checking the answer to be sure that it is right; because engineering and shop calculations *must* be right. The best way to check is to perform the calculations in reverse order, that is from right to left. You can do it and check it with the slide rule in much less time than you can do it longhand.

Not all problems go as smoothly as the foregoing. Occasionally, when you are ready to multiply by the second or third number of the numerator, you will find that this number on the C scale is beyond the end of the D scale. When this happens you must reverse the slide. This you can do by multiplying by one and dividing by one. Multiply by one by setting the hairline of the runner to that index of the C scale which is between the ends of the D scale. Then divide by one by sliding the other index of the C scale under the hairline. Multiplication or division by one does not change the answer; however, it does enable you to continue the process of getting the answer. The following examples demonstrate it.

ILLUSTRATIVE EXAMPLES

4. Calculate $\dfrac{34.8 \times 2.3}{97.5 \times 0.707}$

1) Set the hairline of the runner to the first number of the numerator, *34.8,* on the D scale.
2) Divide by the first number of the denominator, *97.5,* by bringing *97.5* on the C scale under the hairline.
3) You cannot multiply now by the second number of the numerator, *2.3,* because *2.3* on the C scale is beyond the end of the D scale. Hence, you must reverse the slide. Set the hairline of the runner to the right index of the C scale. Leave the runner in this position and bring the left index of the C scale under the hairline.
4) Now multiply by the second number of the numerator, *2.3,* by setting the hairline of the runner to *2.3* on the C scale.
5) Divide by the second number of the denominator, *0.707,* by bringing *0.707* on the C scale under the hairline of the runner.
6) Read the answer, *1.162,* on the D scale under the left index of the C scale.

When the last setting is for a multiplication, read the answer on the D scale under the hairline of the runner. When the last setting is for a division, read the answer on the D scale under the index of the C scale. Obviously, only one index of the C scale can be between the ends of the D scale at one time. The answer is always located on the D scale.

5. Find the result of $\dfrac{385 \times 0.707}{21.6 \times 5.45}$

1) Set the hairline of the runner to the first number of the numerator, *385,* on the D scale.
2) Divide by the first number of the denominator, *21.6,* by sliding *21.6* on the C scale under the hairline.
3) The next step is to multiply by the second number of the numerator, *0.707*. However, *0.707* on the C scale is beyond the end of the D scale. Therefore you must reverse the slide. Place the hairline of the runner over the left index of the C scale. Now move the slide so that the right index of the C scale is under the hairline.
4) Multiply by *0.707* by placing the hairline of the runner on *0.707* on the C scale.
5) Divide by the second number of the denominator, *5.45,* by sliding *5.45* on the C scale under the hairline.
6) Read the answer, *2.31,* on the D scale under the right index of the C scale.

6. Calculate $\dfrac{5.7 \times 1.25 \times 0.375}{8.63 \times 0.283 \times 95}$

1) Set the hairline of the runner to the first number of the numerator, *5.7,* on the D scale.
2) Divide by the first number of the denominator, *8.63,* by bringing *8.63* on the C scale under the hairline.
3) The second number of the numerator, *1.25,* is beyond the left end of the D scale. You must reverse the slide. Therefore, move the hairline of the runner to the right index of the C scale. Then slide the left index of the C scale under the hairline.
4) Multiply by *1.25* by moving the hairline of the runner to *1.25* on the C scale.
5) Divide by the second number of the denominator, *0.283,* by sliding *0.283* on the C scale under the hairline.
6) The third number of the numerator, *0.375,* is beyond the right end of the D scale, so you must reverse the slide. Set the hairline of the runner on the left index of the C scale. Then move the right index of the C scale under the hairline.

7) Now multiply by *0.375* by setting the hairline of the runner to *0.375* on the C scale.
8) Divide by the third number of the denominator, *95,* by sliding *95* on the C scale under the hairline.
9) Read the answer, *0.1152,* on the D scale under the right index of the C scale.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers at the back of the book.

The problems are given to help you acquire skill in manipulation of the slide rule; work only for the sequence of numbers in the answer and do not worry about locating the decimal point.

1. $\dfrac{75 \times 20.2}{32.2} = ?$

2. $\dfrac{44 \times 44}{386 \times 16} = ?$

3. $\dfrac{13 \times 15 \times 17}{12 \times 14 \times 16} = ?$

4. $\dfrac{23 \times 5500 \times 10{,}550{,}000}{648 \times 14{,}000{,}000 \times 226} = ?$

5. $\dfrac{1950 \times 108 \times 32}{11{,}400{,}000 \times 3.14 \times 81} = ?$

6. $\dfrac{98.5 \times 62.4}{2.45 \times 33.1} = ?$

7. $\dfrac{1 \times 1 \times 1}{6.3 \times 5.1 \times 0.133} = ?$

8. $\dfrac{70.7 \times 0.495 \times 8.97}{59.5 \times 2.01} = ?$

9. $\dfrac{3.96 \times 41 \times 2.67}{1.005 \times 6.33 \times 2.18} = ?$

10. $\dfrac{139 \times 22.1 \times 1.708}{65 \times 93.6 \times 72} = ?$

11. $\dfrac{74.2 \times 81.9 \times 69.3}{3.1 \times 2.06 \times 1.55} = ?$

12. $\dfrac{8.66 \times 0.707}{5.77 \times 1.061} = ?$

13. $\dfrac{167000 \times 29 \times 4}{30{,}000{,}000 \times 3.14 \times 8.5} = ?$

14. $\dfrac{2.78 \times 43.2 \times 1.867}{0.798 \times 9.54 \times 8.02} = ?$

15. $\dfrac{555 \times 11.06 \times 4.93}{64.4 \times 1.05 \times 34.6} = ?$

LOCATION OF THE DECIMAL POINT. By this time you should be able to make the slide rule settings for a calculation of this type and obtain the sequence of numbers in the answer. The next thing is to learn how to locate the decimal point in the answer. This is done by making a *digit summation.* There are two stages in this process and they will be designated as A and

B, **A** to be completed during the manipulation of the slide rule, and **B** after the sequence of numbers in the answer has been written.

A. You multiply by each number of the numerator except the first. You divide by each number of the denominator. When multiplying, you set the hairline of the runner to the number on the C scale. When dividing, you slide the number on the C scale under the hairline. Then proceed as follows:

1) When multiplying by a number of the numerator, jot down -1 if the slide projects to the right of the stock. If the slide projects to the left, there is nothing to jot down. There is one exception to this. If you multiply by *1, 10, 100, 1000,* etc., (that is, in a genuine multiplication and not just a reversal of the slide) jot down -1 no matter which way the slide projects.
2) When dividing by a number of the denominator, jot down $+1$ if the slide projects to the right of the stock after the slide is in position. If the slide projects to the left, there is nothing to jot down. There is one exception to this rule. If you divide by *1, 10, 100, 1000,* etc., in a genuine division and not just a reversal of the slide, jot down $+1$ no matter which way the slide projects from the stock.
3) Each reversal of the slide requires multiplication by one, for which you would jot down -1, and division by one, for which you would jot down $+1$. The -1 cancels the $+1$, so do not bother to jot anything down.
4) As you jot down the $+1$'s and -1's, carry them in one horizontal row. A particular problem might yield this:

$$-1+1+1$$

B. After writing down the sequence of numbers in the answer, continue the horizontal row of numbers by adding the digit count* for each number of the numerator and subtracting the digit count for each number of the denominator. The net result is the digit count for the answer. The following examples will illustrate this process. Follow them on your own slide rule and locate the decimal point in the answer.

The horizontal row of numbers will be referred to hereafter as *digit summation,* since it gives the digit count for the answer.

*Digit counts are explained in the chapter on Multiplication, see p. 50.

ILLUSTRATIVE EXAMPLES

7. Calculate $\dfrac{544 \times 12.73}{0.442 \times 3.86}$

Work from left to right, and multiply and divide in alternation.

1) Set the hairline of the runner to *544* on the D scale.
2) Divide by *0.442* by sliding *0.442* on the C scale under the hairline. You are dividing and the slide projects to the right of the stock, so jot down + 1. This is the start of the digit summation.
3) Multiply by *12.73* by setting the hairline of the runner to *12.73* on the C scale. You are multiplying and the slide projects to the right of the stock, so jot down — 1 in the digit summation.
4) Divide by *3.86* by sliding *3.86* on the C scale under the hairline. You are dividing and the slide projects to the left of the stock. Hence, there is nothing to add or subtract from the digit summation.
5) Read the sequence of numbers in the answer as *406*. This is on the D scale under the right index of the C scale.
6) At this point you have in the digit summation,
$$+1-1$$
7) Add the digit count for the first number of the numerator, *544;* that is, add three, and you have,
$$+1-1+3$$
8) Add the digit count for the second number of the numerator, *12.73*. That is, you add two, and the digit summation is,
$$+1-1+3+2$$
9) Subtract the digit count for the first number of the denominator, *0.442*. Since this is zero, you now have,
$$+1-1+3+2-0$$
10) Subtract the digit count for the second number of the denominator, *3.86*. This is one, and the summation is,
$$+1-1+3+2-0-1$$
11) This completes the digit summation. Sum up:
$$+1-1+3+2-0-1=4$$

Then the digit count of the answer is four, so there are four digits to the left of the decimal point in the answer, and it is *4060.*

This may seem long, but the time required to work the problem is much less than that required to tell how to work it. Obviously, you will not need to write down the digit summation each time you add to it or subtract from it. You need to write each number of it only once.

8. Find the result of $\dfrac{21.8 \times 0.577 \times 3980}{1728 \times 3.14 \times 66.5}$

1) Set the hairline of the runner to *21.8* on the D scale.
2) Divide by *1728* by sliding *1728* on the C scale under the hairline. This is a division and the slide projects to the right of the stock so jot down + 1. This is the start of the digit summation.
3) Multiply by *0.577* by setting the hairline of the runner to *0.577* on the C scale. This is a multiplication and the slide projects to the right of the stock so jot down — 1 in the digit summation.
4) Divide by *3.14* by sliding *3.14* on the C scale under the hairline. This is a division and the slide projects to the right of the stock, so add + 1 to the digit summation.
5) Multiply by *3980* by setting the hairline of the runner to *3980* on the C scale. This is a multiplication and the slide projects to the right of the stock so add — 1 to the digit summation.
6) Divide by *66.5* by sliding *66.5* on the C scale under the hairline. This is a division and the slide projects to the right of the stock, so add + 1 to the digit summation.
7) Read the sequence of numbers in the answer as *1390.* This is read on the D scale under the left index of the C scale.
8) At this point, the digit summation should be,
$$+1-1+1-1+1$$
9) Now add to the digit summation the digit count for each number of the numerator. For the numbers *21.8, 0.577* and *3930,* you add, respectively, two, zero, and four. This leaves the digit summation as:
$$+1-1+1-1+1+2+0+4$$

10) Next subtract the digit count for each number of the denominator. For the numbers, *1728, 3.14,* and *66.5,* you subtract, respectively, four, one, and two. This completes the digit summation as,

$$+1-1+1-1+1+2+0+4-4-1-2=0$$

11) Then the digit count for the answer is zero, hence there are zero digits to the left of the decimal point in the answer and it is *0.139.*

All that must be written in doing Example 8 is the problem, the answer, and the digit summation. Thus,

$$\frac{21.8 \times 0.577 \times 3980}{1728 \times 3.14 \times 66.5} = 0.139, \text{ and}$$

$$+1-1+1-1+1+2+0+4-4-1-2=0$$

As soon as you have completed the digit summation, put the decimal point in the answer. The sequence of numbers in the answer should be written as soon as you can read it.

Occasionally when you are ready to multiply by one of the numbers of the numerator, you find that this number on the C scale is outside the D scale. Then you must reverse the slide, that is, put the index of the C scale which is not within the stock in the place of the index which is. Then you can proceed to multiply. The operation of reversing the slide contributes nothing to the digit summation.

ILLUSTRATIVE EXAMPLES

9. Calculate $\dfrac{346 \times 15.62 \times 30.5}{85.5 \times 7.34}$

1) Set the hairline of the runner to *346* on the D scale.
2) Divide by *85.5* by sliding *85.5* on the C scale under the hairline. This is a division and the slide projects to the left of the stock so there is no contribution to the digit summation.
3) The second number of the numerator, *15.62,* is beyond the stock on the C scale. Hence, you must reverse the slide. Bring the hairline of the runner to the right index of the C scale. Then hold the runner fixed while sliding the left index of the

C scale under the hairline. This sort of maneuver contributes nothing to the digit summation.

4) Multiply by *15.62* by setting the hairline of the runner to *15.62* on the C scale. This is a multiplication and the slide projects to the right of the stock, so start the digit summation with — 1.

5) Divide by *7.34* by sliding *7.34* on the C scale under the hairline of the runner. This is a division and the slide projects to the left of the stock so there is nothing to add to the digit summation.

6) Multiply by *30.5* by setting the hairline of the runner to *30.5* on the C scale. This is a multiplication and the slide projects to the left of the stock so there is nothing to add to the digit summation.

7) Read the sequence of numbers in the answer on the D scale under the hairline of the runner. It is read as *2630.*

8) At this point the digit summation should be — 1.

9) Add to the digit summation the digit count for each number of the numerator. For the numbers *346, 15.62* and *30.5,* you add, respectively, three, two, and two. The row of numbers is then,

$$-1+3+2+2$$

10) Subtract the digit count for each number of the denominator. For the numbers *85.5* and *7.34,* subtract, respectively, two and one. This completes the digit summation and it is:

$$-1+3+2+2-2-1=3$$

11) Then the digit count for the answer is three, hence there are three digits to the left of the decimal point in the answer, and it is *263.*

In some cases it is necessary to reverse the slide more than once. However, this adds very little to the amount of work in the problem.

10. Find the result of $\dfrac{3.14 \times 750{,}000 \times 29.3}{1{,}950{,}000 \times 0.462 \times 5.86}$

1) Set the hairline of the runner to *3.14* on the D scale.

2) Divide by *1,950,000* by sliding *1,950,000* on the C scale under the hairline. This is a division and the slide projects to the right of the stock, so start the digit summation with $+1$.

3) The second number in the numerator, *750,000,* on the C scale is beyond the end of the D scale. Hence you must reverse the slide. Move the hairline of the runner to the left index of the C scale and then leave the runner in this position while sliding the right index of the C scale under the hairline. There is no change in the digit summation.

4) Multiply by *750,000* by setting the hairline of the runner to *750,000* on the C scale. This is a multiplication, but since the slide projects to the left of the stock there is nothing to add to the digit summation.

5) Divide by *0.462* by sliding *0.462* on the C scale under the hairline. This is a division, but the slide projects to the left of the stock so it contributes nothing to the digit summation.

6) The third number of the numerator, *29.3,* on the C scale is beyond the end of the D scale, so you must reverse the slide. Do this by setting the hairline of the runner to the right index of the C scale and leaving the runner in this position while bringing the left index of the C scale under the hairline. This adds nothing to the digit summation.

7) Multiply by *29.3* by setting the hairline of the runner to *29.3* on the C scale. The slide projects to the right for this multiplication, so add -1 to the digit summation.

8) Divide by *5.86* by sliding *5.86* on the C scale under the hairline. This is a division and the slide extends to the right of the stock, so add $+1$ to the digit summation.

9) Read the sequence of numbers in the answer as *1304.* This is read on the D scale under the right index of the C scale.

10) At this point you should have in the digit summation,

$$+1-1+1$$

11) Add to the digit summation the digit count for each number of the numerator. For the numbers *3.14, 750,000,* and *29.3,* you add, respectively, one, six, and two, so the digit summation is,

$$+1-1+1+1+6+2$$

12) Subtract from the digit summation the digit count for each number of the denominator. For the numbers *1,950,000*, *0.462*, and *5.86* you subtract respectively, seven, zero, and one. This completes the digit summation as,

$$+1-1+1+1+6+2-7-0-1=2$$

13) Then the digit count for the answer is two, hence there are two digits to the left of the decimal point in the answer and it is *13.04*.

Many calculations involve very small numbers, such as *0.00635*, in which there are several zeros between the decimal point and the first digit of the number. In such a case, remember each zero between the decimal point and the first digit is counted as a negative digit. Example, the digit count for *0.00635* is minus two.

ILLUSTRATIVE EXAMPLES

11. Calculate $\dfrac{0.000781 \times 2730 \times 0.047}{0.0000092 \times 1.54 \times 1.732}$

1) Set the hairline of the runner to *0.000781* on the D scale.
2) Divide by *0.0000092* by sliding *0.0000092* on the C scale under the hairline. In this division, the slide projects to the left of the stock, so there is no contribution to the digit summation.
3) Multiply by *2730* by setting the hairline of the runner to *2730* on the C scale. Since the slide projects to the left of the stock, this gives nothing for the digit summation.
4) Divide by *1.54* by sliding *1.54* on the C scale under the hairline. In this division, the slide projects to the right of the stock, so start the digit summation with $+1$.
5) Multiply by *0.047* by setting the hairline of the runner to *0.047* on the C scale. This is a multiplication and the slide projects to the right, so add -1 to the digit summation.
6) Divide by *1.732* by sliding *1.732* on the C scale under the hairline. In this division, the slide projects to the right of the stock, so add $+1$ to the digit summation.
7) Read the sequence of numbers in the answer on the D scale under the left index of the C scale. It is *408*.

8) At this point the digit summation should be,

$$+1-1+1$$

9) Add to the digit summation the digit count for each number of the numerator. For the numbers *0.000781, 2730,* and *0.047,* these are, respectively, minus three, four, and minus one. Hence the digit summation becomes,

$$+1-1+1-3+4-1$$

10) Complete the digit summation by subtracting the digit count for each number of the denominator. For the numbers *0.0000092, 1.54,* and *1.732,* subtract respectively, minus five, one, and one. This gives finally,

$$+1-1+1-3+4-1-(-5)-1-1=4$$

11) Then the digit count for the answer is four, hence, there are four digits to the left of the decimal point in the answer, and it is *4080.*

12. Calculate $\dfrac{29{,}500{,}000 \times 0.00136 \times 0.0397}{0.00243 \times 7850 \times 0.922}$

1) Set the hairline of the runner to *29,500,000* on the D scale.
2) Divide by *0.00243* by sliding *0.00243* on the C scale under the hairline. In this division, the slide extends to the right so start the digit summation with $+1$.
3) Multiply by *0.00136* by setting the hairline of the runner to *0.00136* on the C scale. This is a multiplication and the slide projects to the right of the stock, so add -1 to the digit summation.
4) Divide by *7850* by sliding *7850* on the C scale under the hairline. This is a division and the slide projects to the left, so there is nothing to add to the digit summation.
5) The third number of the numerator, *0.0397,* on the C scale is beyond the left end of the D scale. Therefore, you must reverse the slide. Move the hairline of the runner to the right index of the C scale. Then, leave the runner fixed and slide the left index of the C scale under the hairline. This adds nothing to the digit summation.
6) Multiply by *0.0397* by setting the hairline of the runner to *0.0397* on the C scale. In this multiplication, the slide projects to the right of the stock so add -1 to the digit summation.

7) Divide by *0.922* by sliding *0.922* on the C scale under the hairline. Since the slide extends to the left there is nothing to add to the digit summation.

8) Read the sequence of numbers in the answer as *906* on the D scale under the right index of the C scale.

9) At this point the digit summation is,

$$+1-1-1$$

10) Add to the digit summation the digit count for each of the numbers of the numerator. For the numbers *29,500,000, 0.00136* and *0.0397,* these are respectively, eight, minus two and minus one. The digit summation becomes,

$$+1-1-1+8-2-1$$

11) Subtract from the digit summation the digit count for each number of the denominator. For the numbers *0.00243, 7850* and *0.922,* you subtract, respectively, minus two, four, and zero. Hence, the digit summation when completed is,

$$+1-1-1+8-2-1-(-2)-4-0=2$$

12) Then the digit count for the answer is two, so there are two digits to the left of the decimal point in the answer, and it is *90.6.*

The next example will show how to apply the rules when there are several numbers such as *1, 10, 100, 1000,* etc. in the problem.

13. Calculate $\dfrac{10 \times 1 \times 100}{0.223 \times 384 \times 1.92}$

When the first number in the numerator is *1, 10, 100, 1000,* etc., always start by setting this number on the left index of the D scale.

1) Set the hairline of the runner to *10* at the left index of the D scale.

2) Divide by *0.223* by sliding *0.223* on the C scale under the hairline. In this division the slide projects to the left beyond the stock, so there is no contribution to the digit summation.

3) Multiply by *1* by setting the hairline of the runner to the right index of the C scale. Whenever you multiply by *1* jot down *— 1* no matter which way the slide projects. This — 1 starts the digit summation.

4) Divide by *384* by sliding *384* on the C scale under the hairline. This is a division and the slide projects to the right of the stock so add + 1 to the digit summation.
5) Multiply by *100* by setting the hairline to the left index of the C scale. No matter which way the slide goes in this multiplication you should add — 1 to the digit summation.
6) Divide by *1.92* by sliding *1.92* on the C scale under the hairline. In this division, the slide extends to the left so there is nothing to add to the digit summation.
7) Read the sequence of numbers in the answer as *609*. This is read on the D scale under the right index of the C scale.
8) At this point the digit summation should be,

$$-1+1-1$$

9) Add to the digit summation the digit count for each number of the numerator. For the numbers *10, 1* and *100,* these are, respectively, two, one and three; so the digit summation becomes,

$$-1+1-1+2+1+3$$

10) Subtract from the digit summation the digit counts for each number of the denominator. For the numbers *0.223, 384* and *1.92,* you subtract respectively, zero, three and one. This completes the digit summation as,

$$-1+1-1+2+1+3-0-3-1=1$$

11) Then the digit count for the answer is one; so there is one digit to the left of the decimal point in the answer, and it is *6.09*.

Some problems contain numbers from which you could cancel common factors. For instance the numerator might contain *81* and the denominator *24,* so you could cancel by dividing each by *3,* leaving *27* and *8,* respectively. Generally, however, it is a waste of time to cancel such numbers. In this case you would still have to set *27* and *8* on the slide rule and this is no easier than *81* and *24.* In most problems you could complete the whole operation on the slide rule with the original numbers in less time than you would take to cancel the common factors. Many such cases arise in which the use of good judgment will save time.

Occasionally the problem will contain more numbers in the denominator than in the numerator, as

$$\frac{1152 \times 6.15}{33.9 \times 0.77 \times 21.8}$$

Here you should insert a one in the numerator so that there will be as many numbers in the numerator as in the denominator. The problem will then be,

$$\frac{1152 \times 6.15 \times 1}{33.9 \times 0.77 \times 21.8}$$

When it is in this form it can be done by the usual methods and the rules for locating the decimal point in the answer will apply. Whether you write down the one or not, you will use it in doing the problem, and if you do not write it down, you may leave it out of the digit summation. It has a digit count of one because it has one digit to the left of the decimal point, and that must be taken into account in locating the decimal point in the answer. If there are several more numbers in the denominator than in the numerator you must insert enough ones in the numerator so that it will contain as many numbers as the denominator. Thus you would change,

$$\frac{554}{3820 \times 0.295 \times 6.17 \times 19.8}$$

to,

$$\frac{554 \times 1 \times 1 \times 1}{3820 \times 0.295 \times 6.17 \times 19.8}$$

before making the calculation.

PRACTICE PROBLEMS

After you have worked all of the problems, check your answers with the correct answers shown at the back of the book.

Check the location of the decimal point as well as the sequence of numbers. Carry out the digit summation for each problem. Do not worry if you miss on the third digit of the answer.

Write down the sequence of numbers in the answer as soon as you can read it. If you wait until you have completed the digit summation, you will have forgotten it and you will have to do the problem over. Also, do not lay the rule down and think that you

will read the answer later. The slide or runner may slip, or you may forget whether to read the answer under the hairline of the runner or under an index of the C scale.

1. $\dfrac{10.66 \times 345}{2.78 \times 65.4} = ?$

2. $\dfrac{100 \times 10 \times 1000}{78 \times 9.6 \times 0.203} = ?$

3. $\dfrac{0.966 \times 0.707 \times 28}{0.577 \times 0.642} = ?$

4. $\dfrac{132000 \times 45.8}{29{,}800{,}000 \times 7.85} = ?$

5. $\dfrac{65 \times 0.0000067 \times 14{,}200{,}000}{7 \times 0.1945} = ?$

6. $\dfrac{1088 \times 576 \times 576}{386 \times 67} = ?$

7. $\dfrac{185 \times 192}{4 \times 0.375} = ?$

8. $\dfrac{23 \times 4980 \times 8000}{648 \times 11{,}400{,}000 \times 115} = ?$

9. $\dfrac{0.123 \times 0.0207 \times 3.24}{1.16 \times 0.201 \times 0.0317} = ?$

10. $\dfrac{34.5 \times 28.7}{16 \times 9.7} = ?$

11. $\dfrac{1.932 \times 0.59 \times 3.14}{0.92 \times 2.17} = ?$

12. $\dfrac{33 \times 33 \times 33}{21 \times 21 \times 21} = ?$

13. $\dfrac{3750 \times 6.28}{60} = ?$

14. $\dfrac{85 \times 5280}{3600} = ?$

15. $\dfrac{10{,}150 \times 24 \times 12}{154 \times 97.5} = ?$

16. $\dfrac{62{,}500 \times 1728}{48 \times 12{,}000{,}000 \times 78} = ?$

17. $\dfrac{0.382 \times 256}{18.5 \times 29} = ?$

18. $\dfrac{78.5 \times 157 \times 22}{85 \times 17 \times 18.3} = ?$

19. $\dfrac{6720 \times 0.0527}{216 \times 428} = ?$

20. $\dfrac{4.95 \times 34.7}{8.6 \times 6.67} = ?$

21. $\dfrac{1260 \times 0.427 \times 21.5}{32.7 \times 516} = ?$

22. $\dfrac{0.693 \times 14}{45.1 \times 9.7 \times 0.23} = ?$

23. $\dfrac{48.4 \times 62.5}{21.2 \times 57.3} = ?$

24. $\dfrac{2.77 \times 0.064}{1.97 \times 0.17} = ?$

25. $\dfrac{3460 \times 84.7}{16{,}000 \times 158} = ?$

26. $\dfrac{0.957 \times 4.15}{1.875 \times 0.015} = ?$

27. $\dfrac{267 \times 17.6}{15.2 \times 576} = ?$

28. $\dfrac{5.25 \times 88 \times 7.5}{22.5 \times 13} = ?$

29. $\dfrac{9600 \times 432}{25{,}600 \times 198} = ?$

30. $\dfrac{0.476 \times 8.43 \times 14.3}{2.75 \times 0.826} = ?$

OTHER TYPES OF SLIDE RULES. The C and D scales are the same on all slide rules. The methods described in this chapter apply to all and should be followed, no matter what kind of slide rule you use.

REVIEW PROBLEMS

Answers to Review Problems are not given in the back of the book. Readers who are working alone may check their answers by working the problems longhand.

1. $\dfrac{35.1 \times 26.2 \times 784}{128 \times 54.2 \times 98} = ?$

2. $\dfrac{7.48 \times 0.407 \times 323}{0.670 \times 92.5 \times 137} = ?$

3. $\dfrac{0.000573 \times 0.072}{0.418 \times 3.14} = ?$

4. $\dfrac{63 \times 56 \times 36}{24 \times 32 \times 15} = ?$

5. $\dfrac{4750 \times 51 \times 51}{32.2 \times 0.611} = ?$

6. $\dfrac{38 \times 38 \times 38}{35 \times 35 \times 35} = ?$

7. $\dfrac{0.1075 \times 23.6}{2730 \times 0.00391} = ?$

8. $\dfrac{0.00058 \times 7130}{0.000082} = ?$

9. $\dfrac{0.01 \times 10 \times 0.001}{11.76 \times 0.0591 \times 0.73} = ?$

10. $\dfrac{3.14 \times 1.826 \times 2.97}{4 \times 0.0342} = ?$

11. $\dfrac{178 \times 0.462}{15.2 \times 0.765} = ?$

12. $\dfrac{6820 \times 4.27}{525 \times 3.68 \times 37.5} = ?$

13. $\dfrac{1 \times 15{,}650 \times 5{,}420{,}000}{48 \times 28{,}900{,}000 \times 278} = ?$

14. $\dfrac{1673 \times 38.7}{17.32 \times 9.63 \times 0.804} = ?$

15. $\dfrac{4.34}{0.707 \times 1.32 \times 2.07} = ?$

CHAPTER VII

THE SQUARE AND SQUARE ROOT

One of the easiest calculations with the slide rule is that of obtaining the square or square root of a number. It is convenient to be able to do this quickly in problems involving areas, solution of quadratic equations, triangles, etc.

THE SQUARE OF A NUMBER. The square of a number is defined as the product of the number multiplied by itself. The proper way to obtain it is (1) to set the hairline of the runner to the number on the D scale and (2) read the square of the number on the A scale under the hairline. A given number can be set in only one location on the D scale so there is no possibility of setting it in the wrong place. (Since the scales on the slide are not used in the calculation, the slide may be in any position.) (3) Locate the decimal.

> **Rule 3. (a) The Square.** *Set the hairline of the runner to the number on the D scale. Read the square of the number on the A scale under the hairline.*

Location of the Decimal Point. The problem of locating the decimal point in a number is just the problem of finding the digit count* of the number, then using it to count off the digits to the left or right of the decimal point, depending upon whether the digit count is plus or minus. You will remember from the discussion of the "Scales of the Slide Rule" that the digits of a number are the numerals of the number, with the limitation that the first digit is the first numeral other than zero. Any other digit than the first can be zero. Thus, in the decimal number *0.00377,*

*For the explanation of Digit Counts, see p. 50.

the first digit is *3* since *3* is the first numeral which is not zero. In the number *28.5,* the first digit is *2,* the second digit is *8,* and the third is *5.* In the case of a number such as *1572* in which no decimal point is shown, it is always understood that the decimal point follows the last numeral. The number *1572* could iust as well be written *1572.0.*

The A scale is a double scale and this fact can be used in locating the decimal point in the square of a number.

Case 1. *When the square of the number is read in the left half of the A scale, multiply the digit count for the number by 2*

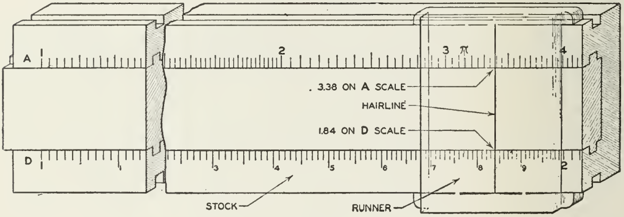


Fig. 28

and subtract 1. The result is the digit count for the square of the number.

Case 2. *When the square of the number is read on the center index or in the right half of the A scale, multiply the digit count for the number by 2. The result is the digit count for the square of the number.* (The center index of the A scale is the number 1 in the center of the scale. It is read as *10, 1000,* etc.)

In the actual procedure of squaring a number, it is best to write down the digits in the square as soon as you can read them; then apply the rules for locating the decimal point. Leave the runner in position until you have placed the decimal point in the answer.

ILLUSTRATIVE EXAMPLES

1. Square 1.84. The correct setting of the runner for this problem is shown in Fig. 28. The steps in the problem are:

1) Set the hairline of the runner to *1.84* on the D scale.
2) Read the digits in the square of *1.84* on the A scale under the hairline. The numerals are *338*. Notice that this is read in the left half of the A scale, so the rule in Case 1 will apply for locating the decimal point.
3) The digit count for *1.84* is one, so multiply one by two, and subtract one from the result in accordance with Case 1. This gives *1*. Then the digit count for the square of *1.84* is one, so the square is 3.38.

2. Square 45.7.

1) Set the hairline of the runner to *45.7* on the D scale.
2) Read the digits in the square as *209*. This is read on the right half of the A scale under the hairline; hence the rule in Case 2 is to be used for locating the decimal point.
3) The digit count for *45.7* is two. Multiply two by two, obtaining four, which in accordance with Case 2, is the digit count of the square; so there are four digits to the left of the decimal point in the square of *45.7* and it is *2090*.

3. There are 320 rods in one mile. How many square rods in a square mile? The answer is the square of *320*.

1) Set the hairline of the runner to *320* on the D scale. The number *320* is between the divisions marked *3* and *4*. (The divisions are the marks designated by the numbers *2, 3, 4, 5, 6, 7, 8* and *9*. Division *2* is about three-tenths of the way from the left index.) From the division marked *3*, count two sections to the right. In this portion of the scale, a section consists of five small spaces. The number *320* is located at the right end of the second section.
2) Read the digits in the square of *320* on the A scale under the hairline. The digits are *102* as far as they can be read. The first digit is *1* since the hairline is between the center index and the division marked *2*. It is in the first section to the right of the center index so the second digit is zero. Each small space has a value of two in the third digit of the number, and since there is only one space between the center index and the hairline, the third digit is *2*.

3) The digit count for *320* is three. Multiply three by two, obtaining six for the result. Since the square of *320* was read in the right half of the A scale, the rule in Case 2 holds; six is the digit count for the answer, so there are six digits to the left of the decimal point in the answer and it is *102,000*. There are *102,000* square rods in one square mile.

4. A square wire is 0.016″ on a side. What is the area of its cross section? The answer is the square of *0.016*.

1) Set the hairline of the runner to *0.016* on the D scale. The first digit of *0.016* is *1* so the number is located between the left index and the division marked *2*. (Notice that this *2* is the second *2* from the left index and is about three-tenths of the length from the left index.) The second digit is *6*, so count six sections from the left index. In this portion of the scale, each section contains ten spaces. Since there are only two digits in the number, it is located at the right edge of the sixth section. The hairline should coincide with the long mark by the number *6*.
2) Read the digits in the square of *0.016* as *256* on the A scale under the hairline. Notice that this is read in the left half of the A scale.
3) The digit count is minus one for *0.016*. Two times minus one is minus two. Since the square of *0.016* was read in the left half of the A scale, the rule in Case 1 for locating the decimal point applies, and you must subtract one from minus two. This gives minus three, which is the digit count for the answer. Hence there are three zeros to the right of the decimal point in the answer and it is *0.000256*. The area of the cross-section of the wire is *0.000256* square inches.

5. Square 0.00579.

1) Set the hairline of the runner to *0.00579* on the D scale. The first digit of *0.00579* is *5* so the number is located between the divisions marked *5* and *6*. Since the second digit is *7*, count seven sections to the right from division *5*. Notice that each section has only two spaces in this portion of the scale. The third digit of the number is *9*, and each space has a value of

five in the third digit, so estimate one and four-fifths spaces from the right of the seventh section. This locates the number.

2) Read the digits in the square of *0.00579* as *336* on the right half of the A scale under the hairline.

3) The digit count is minus two for *0.00579*. Two times minus two is minus four. The square was read in the right half of the A scale, so, there is nothing to subtract from minus four, and this is the digit count for the answer. This means that there are four zeros after the decimal point and before the first digit of the answer. Thus the answer is *0.0000336*.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers in the back of the book.

1. A floor of a square room measures 17.5 feet on each side. Find its area.

2. The area of a circle is 3.14 times the square of its radius. Find the area of a circle of 6.5-inch radius.

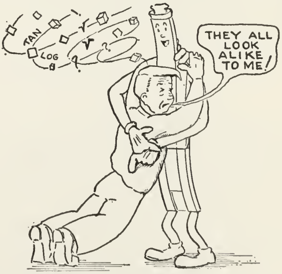
3. One inch is equivalent to 2.54 centimeters. How many square centimeters are there in 1 square inch?

4. The strength of a rectangular beam is proportional to the square of its depth. How much stronger is a rectangular beam 8.3 inches deep than one 3.7 inches deep? Other factors are the same for the two beams. (*Hint:* Divide the square of 8.3 by the square of 3.7.)

5. Steel plate 1 inch thick weighs 0.283 pound per square inch. How much does a square plate 10.6 inches on a side weigh?

Square each of the following numbers:

6. 0.0737
7. 196.7
8. 32.1
9. 0.312
10. 5280
11. 0.00444
12. 8.66
13. 204
14. 0.0371
15. 633



SQUARE AND CUBE ROOTS WILL LOOK ALIKE UNLESS YOU STUDY CAREFULLY

THE SQUARE ROOT OF A NUMBER. The square root of a number is a second number which when multiplied by itself will give the original number. In order to obtain the square root of a number with the slide rule, you would set the hairline of the runner to the number on the A scale and read its square root on the D scale under the hairline.

Since the A scale is a double scale and a given number could be located in either half of it, the question arises as to which half to use in each particular problem. The answer is very simple. If the digit count is an odd number, locate the number in the left half of the A scale. If the digit count is an even number locate the number in the right half of the A scale. One special case must be mentioned. On the A scale there are three points that might be read as *1, 10, 1000,* etc. These points are the left index, the center index and the right index. In taking the square root of such a number, however, the correct index to be used depends upon where the decimal point of the number is located; and you must use the correct index. Numbers such as *10* and *1000* in which the digit count is even, are to be located on the center index of the A scale. When the digit count is odd (for example, 1 or 100) locate it on the left index of the A scale. A few simple examples will show how to deal with such numbers.

> **Rule 3. (b) The Square Root.** *Set the hairline of the runner to the number on the A scale. Read the square root of the number on the D scale under the hairline.*

ILLUSTRATIVE EXAMPLES

6. Where is the number 10 located on the A scale for the purpose of finding the square root?

1) The digit count for *10* is two, and two is an even number so *10* is set on the center index of the A scale. The center index is the mark numbered *1* in the center of the scale.

7. Where is the number 10,000 located on the A scale for the purpose of finding the square root?

1) The digit count is five for *10,000*, and five is an odd number. Hence, *10,000* is located at the left index of the A scale.

Applying the rules to a decimal fraction, such as *0.0000123*, each zero between the decimal point and the first digit of the number is treated as a negative digit, and the digit count is minus four. Negative numbers, like positive numbers, are even or odd. Some decimal fractions, such as *0.707*, have no zeros between the decimal point and the first digit; these numbers are considered to have a digit count of zero. In applying the rules in such cases, zero is considered to be an even number. The following problems will give practice in dealing with numbers less than one.

PRACTICE PROBLEMS

Determine where in the A scale each of the following numbers should be located for the purpose of finding its square root. *After* you have written down which parts of the A scale you would use, check your answers with the correct answers at the back of the book.

1. 0.951
2. 0.0134
3. 0.001
4. 0.000456
5. 0.0001
6. 0.0742
7. 0.1
8. 0.00548
9. 0.0000242
10. 0.01

Location of the Decimal Point. There are two rules for locating the decimal point in the square root of a number.

Case 1. *When the digit count for the original number is odd, add one to the digit count and divide by two. The result gives the digit count for the square root. This procedure is to be followed when the number is located in the left half of the A scale.*

Case 2. *When the digit count for the original number is even, divide the digit count by two. The result gives the digit count for the square root. This is the procedure when the number is located in the right half of the A scale.*

The following examples illustrate the method of finding the square root of a number and the process of locating the decimal point.

ILLUSTRATIVE EXAMPLES

8. Find the square root of 152.

1) The digit count for *152* is three. Three is an odd number, so set the hairline of the runner to *152* in the left half of the A scale.
2) Read the digits in the square root of *152* on the D scale under the hairline. The digits are *1234*.
3) The digit count for *152* is odd, so add one to it, and divide by two, thus: $3 + 1 = 4$ and $4 \div 2 = 2$. Hence there are two digits to the left of the decimal point in the square root of *152* and it is *12.34*.

9. Find the square root of 0.00663.

1) The digit count for *0.00663* is minus two. Minus two is an even number, so set the hairline of the runner to *0.00663* on the right half of the A scale.
2) Read the digits in the square root of *0.00663* as *816* on the D scale under the hairline.
3) The digit count for *0.00663* is minus two. This is even, so divide by two, thus: $-2 \div 2 = -1$. Hence there is one zero to the right of the decimal point in the square root of *0.00663* and it is *0.0816*.

10. Find the square root of 1000.

1) The digit count for *1000* is four, and four is an even number, so set the hairline of the runner to the center index of the A scale.
2) Read the digits in the square root of *1000* as *316*. This is read on the D scale under the hairline.
3) Since the digit count for 1000 is even, divide it by two, thus: $4 \div 2 = 2$. Hence there are two digits to the left of the decimal point in the square root of *1000* and it is *31.6*.

11. Find the square root of 79,600.

1) The digit count for *79,600* is five, and five is an odd number. Therefore, set the hairline of the runner to *79,600* in the left half of the A scale.
2) Read the digits in the square root of *79,600* as *282* on the D scale under the hairline.

3) Since the digit count for *79,600* is odd, add one to the number of digits and divide by two, thus: $5 + 1 = 6$ and $6 \div 2 = 3$. Hence there are three digits to the left of the decimal point in the square root of *79,600* and it is *282*.

12. Find the square root of 0.0327.

1) The digit count is minus one, an odd number for *0.0327,* so set the hairline of the runner to *0.0327* on the left half of the A scale.
2) Read the digits in the square root of *0.0327* on the D scale under the hairline. The digits are *181*.
3) Minus one, the digit count for *0.0327,* is an odd number; therefore, add one and divide by two, thus: $-1 + 1 = 0$ and $0 \div 2 = 0$. Hence there are zero digits to the left of the decimal point in the square root of *0.0327,* and it is *0.181*.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers in the back of the book.

1. A square bin is to have a floor area of 87 square feet. Find the length of one side.

2. There are 43,560 square feet in 1 acre. What is the length of one side of a square plot of ground that is 1 acre in area?

3. What is the radius of a circle that has an area of 3.89 square inches? (*Hint:* Divide 3.89 by 3.14 and take the square root.)

4. Solve for x, in the equation,

$$x^2 = 0.067.$$

5. The area of an equilateral triangle is 0.433 times the square of the length of one side. Find the side of an equilateral triangle which has an area of 6.13 square feet.

Find the square root of each of the following numbers:

6. 22,400,000	**11.** 10
7. 2,240,000	**12.** 0.001
8. 0.000348	**13.** 577,000
9. 793	**14.** 29,500
10. 0.1943	**15.** 0.0000123

SUPPLEMENTARY REMARKS. Finding the square of a number is ordinarily only a part of the problem. Usually

other operations such as multiplication or division form a part of it. You may be tempted to perform these other operations with the A and B scales and of course you could, since each half of the A and B scales offers the same range of numbers as the C and D scales. However, there are two good reasons for not doing this, both depending upon the fact that a given numerical range on the A scale is compressed to half the length that it occupies on the D scale. *First,* you can neither locate nor read a number on the A scale as precisely and quickly as you can on the D scale. *Second,* you are likely to experience eye strain in using the A and B scales for multiplication and division. The time saved by using the A and B scales instead of the C and D scales for multiplication and division is small in comparison with that saved by substituting the slide rule method for the longhand method. This last factor is the important one. To accomplish this saving is a considerable achievement; hence the A and B scales are not recommended for multiplication and division. The small amount of additional time that may be saved does not compensate for possible eye strain and loss of precision.

BASIS OF THE PROCESS. Like other slide-rule calculations, finding the square of a number is based on logarithms. The square of a number is the product of the number multiplied by itself. Hence the logarithm of the square of a number is equal to the logarithm of the number plus the logarithm of the number, or two times the logarithm of the number. By way of example, let us find the square of *15.62* by use of logarithms. From a table of logarithms* to the base 10,

$$\text{Logarithm of } 15.62 = 1.1937$$
$$2 \times 1.1937 = 2.3874$$

The logarithm of the square of *15.62* is 2.3874. In a table of logarithms, *2.3874* is found to be the logarithm of *244;* then the square of *15.62* is *244.* You will recall that there are two parts to a logarithm, the *characteristic,* or part to the left of the decimal point, and the *mantissa,* or part to the right of the decimal point. The characteristic of the logarithm of a number depends on the location

*The log scale of the slide rule can be used; however, only three digits can be read accurately.

of the decimal point of the number and is one less than the digit count for the number. The mantissa of the logarithm of a number depends only on the sequence of numbers and is independent of the location of the decimal point of the number. Only the mantissa of the number is represented on the slide rule. The distance from the left index of the D scale to the location of a number represents the mantissa of the logarithm of the number. Since a numerical range on the A scale occupies only half the length that it would on the D scale, the same number, say *2,* is only half as far from the left index of the A scale as from the left index of the D scale. Conversely, at the same distance from the left index, the number on the A scale has a mantissa twice as large as the number on the D scale. Hence, the number on the A scale is the square of the number directly below it on the D scale.

Location of the Decimal Point by Logarithms. If the mantissa of the logarithm of a number is less than *0.5000,* there is no "carryover" to the characteristic column in multiplying by two. Hence, the characteristic of the square of the number is just twice the characteristic of the number itself. This is the case when the square is read in the left half of the A scale. With the characteristic of the square of the number known, the decimal point can be located, since the characteristic is one less than the digit count. However, it is more convenient to have the relation directly in terms of digits. As an equation the characteristic relation is,

$$2 \times \text{characteristic of number} = \text{characteristic of square}$$

When this is put in terms of digits, it is:

$$2\ (\text{digit count for number} - 1) = \text{digit count for square} - 1$$

or

$$2 \times \text{digit count for number} - 2 = \text{digit count for square} - 1$$

This reduces to,

$$2 \times \text{digit count for number} - 1 = \text{digit count for square}$$

In words, if the square is read on the left half of the A scale, the digit count for the square of a number is one less than twice the digit count for the number.

If the mantissa of the logarithm of a number is more than *0.5000,* there is a "carryover" of one to the characteristic column in multiplying by two. Therefore, the characteristic of the square

of the number is one more than twice the characteristic of the number. This is the case when the square is read on the right half of the A scale. The equation which expresses the relation between characteristics is,

$$2 \times \text{characteristic of number} + 1 = \text{characteristic of square}$$

When this is expressed in terms of digits, it becomes,

$$2(\text{digit count for number} - 1) + 1 = \text{digit count for square} - 1$$

or

$$2 \times \text{digit count for number} - 2 + 1 = \text{digit count for square} - 1$$

This reduces to,

$$2 \times \text{digit count for number} = \text{digit count for square}$$

That is, when the square is read in the right half of the A scale, the digit count for the square of a number is exactly twice the digit count for the number.

These results can be used to show where to locate a number on the A scale in order to read its square root on the D scale. If the square is located in the left half of the A scale,

$$2 \times \text{digit count for number} - 1 = \text{digit count for square}$$

or

$$\text{digit count for number} = \frac{\text{digit count for square} + 1}{2}$$

This can be rewritten for purposes of studying the square root. The relation of a number to its square is the same as the relation of the square root of a given number to the given number. Hence,

$$\text{digit count for square root} = \frac{\text{digit count for given number} + 1}{2}$$

The digit count for the square root must be an integer. Hence, the digit count for the given number must be an odd number so that when added to one and divided by two the result will be an integer. Conversely, if the digit count for a given number is odd, the number must be located in the left half of the A scale so that its square root can be read directly below on the D scale.

If the square of a number is located in the right half of the A scale,

$$2 \times \text{digit count for number} = \text{digit count for square}$$

or

$$\text{digit count for number} = \frac{\text{digit count for square}}{2}$$

This can be rewritten just as in the preceding case.

$$\text{digit count for square root} = \frac{\text{digit count for given number}}{2}$$

Again, the digit count for the square root must be an integer. Thus the digit count for the given number must be even, so that when it is divided by two the result will be an integer. Therefore, if the digit count for a given number is even, the number must be located in the right half of the A scale so that the square root can be read directly below on the D scale.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers at the back of the book.

Square each of the following numbers:

1. 23
2. 1.954
3. 6.17
4. 384
5. 0.878
6. 5280
7. 41.7
8. 0.0718
9. 12.7
10. 56.5

Find the square root of each of the following numbers:

11. 1230
12. 93.5
13. 378
14. 2420
15. 163,000
16. 0.082
17. 1.67
18. 0.733
19. 21.5
20. 848

Solve the following:

21. $\sqrt{(24.5)^2 + (33.1)^2} = ?$
22. $(1.75)^2 + 2.92 = ?$
23. $\sqrt{3100 + (88)^2} = ?$
24. $\sqrt{(382)^2 + (256)^2} = ?$
25. $\sqrt{(9.43)^2 + (5.85)^2} = ?$

OTHER TYPES OF SLIDE RULES. All calculations for square and square root can be made with the A and D scales. These are located the same on all slide rules. Hence, you can use the methods described in this chapter for all slide rules.

REVIEW PROBLEMS

Answers to the Review Problems are not given in the back of the book. Readers who are working alone may check their answers by working the problems longhand.

Some of these Review Problems are more comprehensive than the earlier ones in the chapter and require other operations in addition to finding square and square root; however, most practical problems do.

1. The square of the hypotenuse of a right triangle is equal to the sum of the squares of the legs. Find the hypotenuse of a right triangle which has legs of 6 inches and 9 inches, respectively.

2. Solve for x.

$$x = \frac{13 + \sqrt{(13)^2 - 96}}{3}$$

3. The hypotenuse of a certain right triangle is 8.5 inches and one leg is 7.3 inches. Find the other leg.

4. Calculate $\frac{\pi}{4}(2.25)^2$ (Hint: $\pi = 3.14$)

5. On which scale do you locate a number when you want to square it?

6. The volume of a right circular cylinder is equal to 3.14 times the square of the radius times the altitude. Find the volume of a right circular cylinder which has a radius of 1.75 inches and an altitude of 2.75 inches.

7. The volume of a right circular cone is one-third times 3.14 times the square of the radius of the base times the altitude. Find the volume of a right circular cone which has a radius of 0.63 inches and an altitude of 0.31 inches.

8. On which scale is the square of a number read?

9. It is desired to build a cylindrical tank, 10 feet in diameter, to have a capacity of 1800 cubic feet. What altitude should the tank have?

10. Steel weighs 0.283 pounds per cubic inch. Find the weight of a circular steel bar 2.5 inches in diameter and 21 inches long.

11. The area of a circle is equal to $\frac{\pi}{4}$ d^2 where d is the diameter. Find the area of a circle of 118-foot diameter.

12. What is the diameter of a circular wire that has a cross-sectional area of 0.013 square inches?

13. Solve for x.

$$x = \sqrt{(21.5)^2 - (14.3)^2}$$

14. Find the length of the diagonal of a rectangle which has sides of 27 and 33 feet, respectively.

15. A cylindrical tank is 60 feet in diameter and 36 feet high. How many gallons of water will it hold? One cubic foot is equivalent to 7.48 gallons.

In each of the following problems, perform the indicated operation.

16. $(384)^2$

17. $\sqrt{1942}$

18. $\sqrt{(138)^2 + (87.5)^2}$

19. $(0.0756)^2$

20. $(66.8)^2$

21. $(1.972)^2$

22. $\sqrt{3030}$

23. $(0.00373)^2$

24. $\sqrt{0.00373}$

25. $\sqrt{108{,}000{,}000}$

CHAPTER VIII

THE CUBE AND CUBE ROOT

Many formulae require the use of the cube or cube root of a number. These operations are so easily and quickly done with the slide rule that everyone who owns one should know the processes.

The easiest way to find the cube or cube root of a number is to use the K scale with the D scale by a process in which no other scales are used. However, many slide rules do not have a K scale. For this reason, this chapter is divided into two parts, Part A in which the methods which do not use the K scale are explained, and Part B in which the use of the K scale is explained. If your slide rule has a K scale, by all means use the method of Part B.

PART A—CUBE AND CUBE ROOT WITHOUT THE K SCALE

THE CUBE OF A NUMBER. The cube of a number is defined as the product of the number multiplied by itself twice. That is, the cube of a number a is equal to $a \times a \times a$. An easy way to regard this calculation is to consider it as the number multiplied by its square. You already know how to square a number. After obtaining its square, then just multiply the number by the square.

STEPS IN THE PROCESS OF FINDING THE CUBE OF A NUMBER

You can carry out the process in the following steps:

1) Set the hairline of the runner to the number on the D scale. Notice that the square of the number is located on the A scale under the hairline.

2) Leave the runner in position and bring the proper index of the C scale under the hairline to multiply by the square of the number. At this stage, the square of the number is still located on the A scale under the hairline.

3) Read the square of the number on the A scale under the hairline of the runner and then set the hairline to the square on the C scale.

4) Read the cube of the number on the D scale under the hairline.

A few examples will serve to illustrate this process. For the present we will concentrate on getting the digits in the cube. Later, the problem of locating the decimal point will be considered.

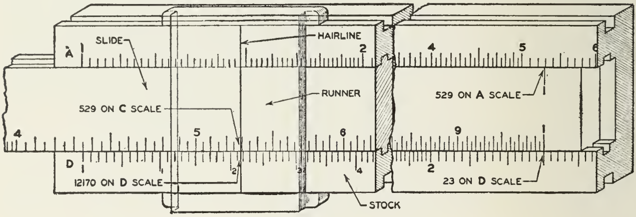


Fig. 29

ILLUSTRATIVE EXAMPLES

1. Cube 23. Fig. 29 shows the proper setting of the slide and runner for this operation.

1) Set the hairline of the runner to *23* on the D scale.
2) Slide the right index of the C scale under the hairline.
3) Notice that the square of *23* is *529*. Then set the hairline of the runner to *529* on the C scale.
4) Read the answer, *12170,* on the D scale under the hairline.

In the second step of this process, it is necessary to place the proper index of the C scale under the hairline of the runner. If you cannot multiply by the square of the number when the right index is under the hairline, then you must reverse the slide and place the left index under the hairline.

2. Cube 125.

1) Set the hairline of the runner to *125* on the D scale.
2) Slide the left index of the C scale under the hairline.
3) Read the square of *125* as *15600* on the A scale under the hairline of the runner and then set the hairline to *15600* on the C scale.
4) Read the answer, *1948000* on the D scale under the hairline.

Do not worry now about the location of the decimal point in the cube of the number. After you know how to find the digits in the cube, you can study the method for locating the decimal point.

3. Cube 71.

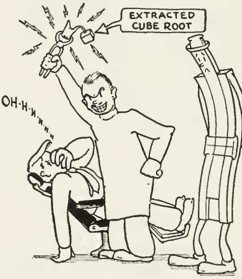
1) Set the hairline of the runner to *71* on the D scale.
2) Slide the right index of the C scale under the hairline.
3) Read the square of *71* as *5040* and set the hairline of the runner to *5040* on the C scale.
4) Read the answer, *358000,* on the D scale under the hairline.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

Cube each of the following numbers:

1. 15
2. 27
3. 8
4. 7
5. 57
6. 19
7. 42
8. 9
9. 99
10. 105



STUDY MAKES EXTRACTION EASY

Location of the Decimal Point. In the foregoing examples no attention was paid to locating the decimal point in the cube of the number. However, most problems of finding the cube involve numbers in which it would be difficult to locate the decimal point by inspection. Hence, it is desirable to have a procedure for

locating the decimal point in the cube of a number. This procedure is based upon:

A. Whether the square of the number is read on the left half or right half of the A scale.

B. Whether the slide projects to right or left in multiplying the number by its square.

The procedure is:

1) Multiply the digit count* for the number to be cubed by three. In any number such as *227* where no decimal point is shown, it is always understood that the decimal point is just after the last digit. The number *227* could just as well be written as *227.00*.
2) Subtract one from this result if the square of the number is read on the left half of the A scale. If read on the right half, there is nothing to add or subtract.
3) Subtract one more if the slide projects to the right of the stock when you multiply the number by its square. If the slide projects to the left there is nothing to add or subtract.
4) The final result is the digit count for the cube of the number.

Leave the slide in position while you write down the digits in the answer. Then think about locating the decimal point. The square of the original number is located on the A scale directly above that index of the C scale which is between the ends of the D scale, so you know in which half it lies. Also, you can see by inspection which way the slide projects. A few examples will demonstrate the procedure.

ILLUSTRATIVE EXAMPLES

4. Cube 32.7.

The digits in the answer are read as *35*. The *3* and the *5* are the only digits other than zeros that you can read in this answer. After you find the digit count for the answer you can add whatever zeros are necessary.

1) The digit count for *32.7* is two. Multiply two by three. The result is six.

---

*For the explanation of Digit Counts, see p. 50.

2) The square of *32.7* is located on the A scale directly above the left index of the C scale. Since it is in the right half of the A scale there is nothing to add to or subtract from six.
3) The slide projects to the right of the stock; hence, subtract one from six. The result is five.
4) The digit count for the cube of *32.7* is five, and it is *35000*.

Remember that in numbers less than one, each zero between the decimal point and the first digit is considered as a negative digit; which means that the digit count is negative. The process of locating the decimal point in the cube of such numbers is shown in Example 5.

5. Cube 0.0165. The digits in the cube of *0.0165* are read as *448*.

1) The digit count for *0.0165* is minus one. Multiply minus one by three. The result is minus three: $-1 \times 3 = -3$.
2) The square of *0.0165* is located in the left half of the A scale, so subtract one from minus three. The result is minus four. When you subtract from a negative number, the result is farther from zero than the original number. Thus when one is subtracted from minus* three, the result is minus four: $-3 - (+1) = -4$.
3) Since the slide extends to the right of the stock in multiplying *0.0165* by its square, subtract one from minus four. This gives minus five: $-4 - (+1) = -5$.
4) There are minus five digits in the cube of *0.0165*. As the final digit calculation shown in step 3 gives minus five as the digit count for the answer, you place five zeros to the right of the decimal point, between the decimal point and the first digit of the answer. Hence the answer is 0.00000448.

6. Cube 624. The digits in the cube of *624* are read as *242*.

1) The digit count for *624* is three. Three multiplied by three is nine.
2) The square of *624* is read on the right half of the A scale. Therefore, there is nothing to add to or subtract from nine at this point.

---

*An explanation of negative numbers is given on p. 240.

3) The slide projects to the left of the stock, so nine stands unchanged and is the digit count for the answer.
4) There are nine digits, then, to the left of the decimal point in the cube of *624* and it is *242,000,000*. You can read only the first three of these digits on the slide rule. Consequently, it is necessary to add the six zeros to bring the total up to nine. Whenever the digit count is positive, any additional zeros are inserted to the left of the decimal point.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

Find the cube of each of the following numbers:

1. 8.5	**3.** 0.0579	**6.** 0.286	**9.** 2.16
2. 123	**4.** 44.3	**7.** 11.05	**10.** 0.00343
	5. 3.75	**8.** 0.93	

11. Find the volume of a cube that is 1.56 inches on a side.

12. The volume of a sphere is four-thirds times 3.14 times the cube of the radius. Find the volume of a sphere that has a radius of 0.667 inches.

13. Solve for x,

$$x = (13.5)^3 + 2.8(9.4)^3$$

14. Find the volume of a sphere of 11-foot radius.

15. Find the cube of $\dfrac{32.8 \times 56.5}{781}$

BASIS OF THE PROCESS OF FINDING THE CUBE. The process of finding the cube of a number resolves into finding the square and multiplying it by the number. Each of these sub-processes has been discussed in an earlier chapter, so no further comments are necessary.

Rules for locating the decimal point simply combine those rules which apply to finding the square and to multiplication.

THE CUBE ROOT OF A NUMBER. The cube root of a number, a, is defined as a second number, b, which, when twice multiplied by itself is equal to a. Thus,

$$a = b \times b \times b$$

The problem is to find b when a is known. The cube root can be

found readily with the slide rule, and a great deal of time is saved over doing it by a longhand process.

DIGITS AND REMAINDERS. Part of the process of finding the cube root of a number depends upon the digit count* for the number, so it will be desirable to consider this subject briefly. The digit count for a number must be an integer, that is a whole number. For instance, the digit count for 1658.5 is four. During the process of finding the cube root of a number, it will be necessary to divide the digit count for the number by three. When this integer is divided by three, the result is one of three things: An integer, an integer plus one-third, or an integer plus two-thirds. Thus if five is divided by three, the result is one (an integer) plus two-thirds. The amount over and above an integer in the quotient will be referred to hereafter as the *remainder*. The remainder is zero, one-third or two-thirds, respectively, when the quotient is an integer, an integer plus one-third, or an integer plus two-thirds. It is necessary for you to be able to find this remainder quickly for any number. Try it in the following examples and problems, but remember that the quantity to be divided by three is the digit count for the number of which you want the cube root.

ILLUSTRATIVE EXAMPLES

7. Find the remainder for the number 15.62.

1) The digit count for *15.62* is two.
2) Divide two by three. The result is two-thirds, which can be read as zero plus two-thirds. Zero is to be considered as an integer so the remainder is two-thirds.

8. Find the remainder for the number 4356.8.

1) The digit count for *4356.8* is four.
2) Divide four by three. The result is one and one-third so the remainder is one-third.

9. Find the remainder for the number 500,000.0.

1) The digit count for *500,000.0* is six.
2) Divide six by three. The result is two, an integer. Therefore, we may consider the remainder as zero.

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*An explanation of digit count is given on page 50.

PRACTICE PROBLEMS

After you have worked all of these problems, check your answers with the correct answers shown at the back of the book.

Find the remainder for each of the following numbers. The remainder is zero or the fraction remaining when the digit count is divided by three.

1. 38.5	3. 483.2	**5.** 0.785
2. 1.58	4. 27500	6. 5280

Some care is necessary in finding the remainder when the digit count is negative. In such a number as *0.00348,* each zero between the decimal point and the first digit of the number is counted as a negative digit, thus the digit count is minus two for *0.00348.* When this number is divided by three, the result must be expressed with a positive remainder. Minus two divided by three is minus two-thirds, but when this is written with a positive remainder, it is minus one plus one-third. Minus one is considered the integer, or whole number, and plus one-third is the positive remainder. A few examples will illustrate this.

ILLUSTRATIVE EXAMPLES

10. Find the remainder for the number 0.0952.

1) The digit count for *0.0952* is minus one.
2) Divide minus one by three. The result is minus one-third, but when it is written with a positive remainder it is minus one plus two-thirds. Hence the remainder is two-thirds.

11. Find the remainder for the number 0.000478.

1) The digit count for *0.000478* is minus three.
2) Divide minus three by three. The result is minus one, which is minus one plus zero. Thus the remainder is zero.

12. Find the remainder for the number 0.00000620.

1) The digit count for *0.00000620* is minus five.
2) Divide minus five by three. The result is minus one and two-thirds, but when this is written with a positive remainder it is minus two plus one-third. The remainder is one-third.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

Find the remainder for each of the following numbers:

1. 0.144	**3.** 0.000718	**5.** 0.000025
2. 0.00577	**4.** 0.01812	**6.** 0.0080

THE PROCESS OF FINDING THE CUBE ROOT. The first two steps in finding the cube root of a number are always the same. They are:

1) Set the hairline of the runner to the number in the right half of the A scale. That is, set the hairline to the number for which you want to find the cube root.
2) Make a digit count and divide it by three. The quotient must be an integer plus a positive remainder. Determine this remainder as zero, one-third, or two-thirds.

The next step depends upon the value of the remainder.

When the Remainder Is Zero. When the remainder is zero, the third step is:

3) Adjust the slide so that the number on the right half of the B scale that is under the hairline is the same as the number on the D scale that is under the right index of the C scale. The number on the D scale that is under the right index of the C scale is the cube root of the original number. Try it in the following examples:

ILLUSTRATIVE EXAMPLES

13. Find the cube root of 216. Fig. 30 shows the setting of the slide and runner for this problem.

1) Set the hairline of the runner to *216* in the right half of the A scale.
2) Make the digit count for *216* and divide it by three. There are three digits to the left of the decimal point in *216*. Dividing three by three gives one with no remainder. That is, the remainder is zero.
3) Adjust the slide so that the number on the right half of the B scale that is under the hairline is the same as the number on the

D scale that is under the right index of the C scale. This number is *6* so the cube root of *216* is *6.*

Do not worry now about locating the decimal point in the cube root; concentrate on finding the digits in the cube root. You can study the method of locating the decimal point later.

14. Find the cube root of 125,000.

1) Set the hairline of the runner to *125,000* in the right half of the A scale.
2) The digit count for *125,000* is six, and six divided by three is two with no remainder. Hence the remainder is zero.

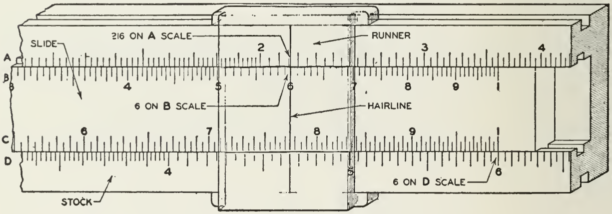


Fig. 30

3) Adjust the slide so that the number in the right half of the B scale that is under the hairline is the same as the number on the D scale that is under the right index of the C scale. This number is *50.* Therefore, *50* is the cube root of *125,000.*

Be sure that you start by locating the number on the right half of the A scale. Do not locate it on the B scale.

15. Find the cube root of 0.729.

1) Set the hairline of the runner to *0.729* in the right half of the A scale.
2) The digit count for *0.729* is zero. Zero divided by three is zero with no remainder. That is, the remainder is zero.
3) Adjust the slide so that the number in the right half of the B scale that is under the hairline is the same as the number on the D scale that is under the right index of the C scale. This number is *0.9,* so *0.9* is the cube root of *0.729.*

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

Find the cube root of each of the following numbers. If it takes four or five seconds, or even ten, to do one of them, think how much longer it would take to do it longhand.

1. 0.123	**3.** 950,000	**6.** 0.750	**9.** 441
2. 188	**4.** 525	**7.** 288	**10.** 636
	5. 315	**8.** 863	

When the Remainder Is One-Third. Next we consider the case when the remainder is one-third after you have divided the digit count by three. In this case the third step is:

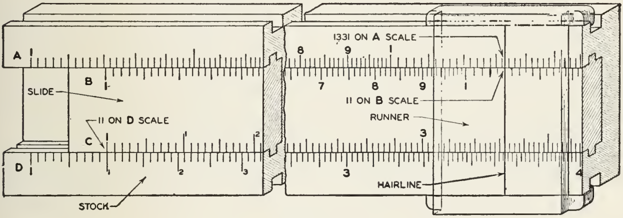


Fig. 31

3) Adjust the slide so that the number on the right half of the B scale that is under the hairline is the same as the number on the D scale that is under the left index of the C scale.

Remember the first and second steps are always the same.

ILLUSTRATIVE EXAMPLES

16. Find the cube root of 1331. The proper setting for the slide and runner is shown in Fig. 31.

1) Set the hairline of the runner to *1331* on the right half of the A scale.
2) The digit count for *1331* is four. Divide four by three. The result is one and one-third, so the remainder is one-third.

3) Adjust the slide so that the number on the right half of the B scale that is under the hairline is the same as the number on the D scale that is under the left index of the C scale. This number is *11,* so *11* is the cube root of *1331.*

It is necessary to carry out the second step in each problem so that you will know which procedure to use in the third step.

17. Find the cube root of 3375.

1) Set the hairline of the runner to *3375* in the right half of the A scale.
2) The digit count for *3375* is four. Four divided by three is one and one-third. Thus the remainder is one-third.
3) Adjust the slide until the number on the right half of the B scale that is under the hairline is the same as the number on the D scale that is under the left index of the C scale. This number is *15.* Therefore, *15* is the cube root of 3375.

18. Find the cube root of 0.008.

1) Set the hairline of the runner to *0.008* in the right half of the A scale.
2) The digit count is minus two for 0.008. Divide minus two by three; the result is minus two-thirds. This must be rewritten as minus one plus one-third so as to have a positive remainder. The remainder is one-third.
3) Adjust the slide so that the number on the right half of the B scale that is under the hairline is the same as the number on the D scale that is under the left index of the C scale. This number is *0.2,* so the cube root of *0.008* is *0.2.*

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

Find the cube root of each of the following numbers:

1. 1.55	**3.** 1222	**6.** 9.22	**9.** 3.14
2. 5.43	**4.** 6.93	**7.** 1.12	**10.** 8.36
	5. 3.75	**8.** 4500	

When the Remainder Is Two-Thirds. In the second step in the process of finding the cube root of a number, you divide

the digit count by three. The result of the division is an integer plus a remainder. If the remainder is two-thirds, the third step is:

3) Adjust the slide so that the number on the left half of the B scale that is under the hairline of the runner is the same as the number on the D scale that is under the left index of the C scale. This number on the D scale under the left index of the C scale is the cube root of the original number.

ILLUSTRATIVE EXAMPLES

19. Find the cube root of 64. The proper positions for the slide and runner are shown in Fig. 32. The procedure in doing the problem is:

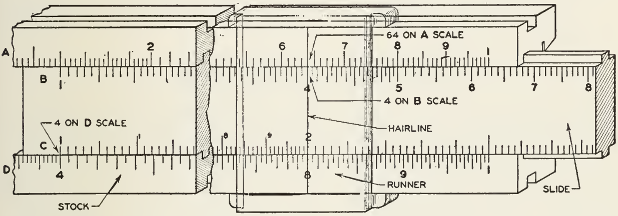


Fig. 32

1) Set the hairline of the runner to *64* on the right half of the A scale.
2) Make a digit count for 64. This is two. Divide two by three, and obtain two-thirds as the result. The remainder is two-thirds.
3) Adjust the slide so that the number on the left half of the B scale that is under the hairline is the same as the number on the D scale that is under the left index of the C scale. This number is *4,* and *4* is the cube root of *64.*

20. Find the cube root of 12200.

1) Set the hairline of the runner to *12200* on the right half of the A scale.
2) The digit count for *12200* is five. Five divided by three is one and two-thirds. The remainder is two-thirds.

3) Adjust the slide so that the number on the left half of the B scale that is under the hairline is the same as the number on the D scale that is under the left index of the C scale. This number is *23*. Hence, *23* is the cube root of *12200*.

21. Find the cube root of 0.027.

1) Set the hairline of the runner to *0.027* on the right half of the A scale.
2) The digit count for *0.027* is minus one. Minus one divided by three is minus one-third. In order to have a positive remainder, this must be rewritten as minus one plus two-thirds. The remainder is two-thirds.
3) Adjust the slide so that the number on the left half of the B scale that is under the hairline is the same as the number on the D scale that is under the left index of the C scale. This number is *0.3* so *0.3* is the cube root of *0.027*.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

Find the cube root of each of the following numbers:

1. 13.5	**3.** 0.024	**6.** 67.5	**9.** 11,000
2. 52.7	**4.** 0.095	**7.** 0.0731	**10.** 88.8
	5. 83,200	**8.** 49.7	

SUMMARY. A brief review of the method of finding the cube root of a number is desirable here. The first two steps are always the same:

1) Set the hairline of the runner to the number in the right half of the A scale.
2) Make a digit count for the original number and divide digit count by three. Express the result as an integer *plus* a remainder. The value of the remainder, whether zero, one-third, or two-thirds, determines what to do in the third step.
3) Adjust the slide so that a certain number on the B scale under the hairline is the same as that on the D scale under one index of the C scale. The following table shows which half of the B scale and which index of the C scale to use for each possible remainder.

TABLE III

Remainder	Half of B Scale	Index of C Scale
Zero	Right	Right
One-third	Right	Left
Two-thirds	Left	Left

Location of the Decimal Point. Now that you know how to find what digits are in the cube root of a number, you are ready to learn how to locate the decimal point. The first step is to divide the digit count for the original number by three. If the result is an integer, this integer is the digit count for the cube root. If the result is not an integer, increase it by just enough to make it an integer. Thus if it is one and two-thirds, increase it to two. After it has been increased to an integer, this integer is the digit count for the cube root.

ILLUSTRATIVE EXAMPLES

22. Find the cube root of 2070.

1) Set the hairline of the runner to *2070* on the right half of the A scale.
2) Make the digit count for *2070* and divide by three. There are four digits to the left of the decimal point and four divided by three is one and one-third. The remainder is one-third so you will use the right half of the B scale and the left index of the C scale in the third step.
3) Adjust the slide so that you have the same number under the hairline on the right half of the B scale that you have on the D scale under the left index of the C scale. The digits in this number are *1274*.
4) The result of dividing the digit count for *2070* by three is one and one-third. Increase this result to the next larger integer, which is two. Then there are two digits to the left of the decimal point in the answer and it is 12.74.

It is necessary to be careful when finding the cube root of a number less than one. In the number 0.00465, for instance, the digit count is minus two, since each zero between the decimal point and the first digit of the number is counted as a negative

digit. The negative number can be divided by three and the result of dividing minus two by three is minus two-thirds. This would have to be rewritten as minus one plus one-third in order to have a positive remainder. However, increasing a negative number to the next larger integer must be done with care. To increase a negative number is always to bring it closer to zero. Thus the next larger integer from minus one plus two-thirds is zero. Similarly, the next larger integer from minus two plus two-thirds is minus one.

23. Find the cube root of 0.0000465.

1) Set the hairline of the runner to *0.0000465* on the right half of the A scale.
2) Make a digit count for *0.0000465*. Each zero between the decimal point and the first digit of the number is counted as a negative number. Hence the digit count is minus four. Divide minus four by three, obtaining minus one and one-third for the answer. You must express this with a positive remainder, so change it to minus two plus two-thirds. The remainder is two-thirds, so you will use the left half of the B scale and the left index of the C scale in the third step.
3) Adjust the slide so that the number on the left half of the B scale that is under the hairline is the same as the number on the D scale that is under the left index of the C scale. Read the digits in this number as *36*.
4) The result of dividing minus four, the digit count for *0.0000465*, by three was minus two plus two-thirds. Increase this to the next larger integer which is minus one. There is one minus digit in the cube root of *0.0000465*, so the cube root is *0.036*. The zero between the decimal point and the *3* is counted as a negative digit.

24. Find the cube root of 0.275.

1) Set the hairline of the runner to *0.275* on the right half of the A scale.
2) The first digit of *0.275* is *2*, so the digit count is zero. Zero divided by three is zero with no remainder. Then the re-

mainder is zero, so you will use the right half of the B scale and the right index of the C scale in the third step.

3) Adjust the slide so that the number on the right half of the B scale under the hairline is the same as the number on the D scale under the right index of the C scale. Read the digits of this number as *65*.

4) The digit count for *0.275* is zero, and zero divided by three is zero. We regard zero as an integer, so it stands unchanged as the digit count for the answer. The cube root of *0.275* is *0.65*.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

1. A certain cube has a volume of 6.3 cubic inches. Find the length of one side.

2. The volume of a sphere is one-sixth times 3.14 times the cube of the diameter. What must be the diameter of a sphere to have a volume of 3000 cubic feet?

3. Solve for x.

$$x^3 = (22.7)^2 + 33$$

Find the cube root of each of the following numbers:

4. 4380	**7.** 6.78	**10.** 27,600	**13.** 535
5. 0.195	**8.** 0.0923	**11.** 0.0000306	**14.** 7.43
6. 183	**9.** 132,000	**12.** 0.0047	**15.** 11,050

Find the cube of each of the following numbers:

16. 2.72	**18.** 0.717	**21.** 0.093	**24.** 30.3
17. 95.7	**19.** 5.61	**22.** 1.27	**25.** 4.7
	20. 15.2	**23.** 8.15	

Find the cube root of each of the following numbers:

26. 37.5	**28.** 0.684	**31.** 75,000	**34.** 1220
27. 427	**29.** 0.0783	**32.** 6.08	**35.** 38,000
	30. 5280	**33.** 192	

Work the problems at the end of the chapter also.

OTHER TYPES OF SLIDE RULES. The processes described so far in this chapter require the use of the A, B, C,

and D scales. These scales are the same on all slide rules. Hence, you can use these methods on any slide rule. However, if your slide rule has a K scale, use the method explained in Part B of this chapter.

BASIS OF THE PROCESS OF FINDING THE CUBE ROOT. The basis of the process can be explained most easily by using a numerical example. You should follow the operation on your own slide rule. Consider the cube root of *500*, which is *7.94*. When the runner and slide are set correctly for this problem, the hairline of the runner is on *500* on the right half of the A scale. Also, *7.94* on the right half of the B scale is under the hairline, and *7.94* on the D scale is under the right index of the C scale. The A and B scales are set to divide *500* by *7.94* with the result on the A scale over the right index of the B scale. The result of dividing the number by its cube root is the square of the cube root, so you have the square of the cube root on the A scale over the right index of the B scale. The number on the D scale under the right index of the C scale is the square root of the number directly above it on the A scale so it must be the cube root of *500*. The process can be verified for any number whatever by a similar examination.

Location of Decimal Point by Logarithms. The rule for locating the decimal point in the cube root can be justified by logarithms. The logarithm of the cube of a number is three times the logarithm of the number. Conversely, the logarithm of the cube root of a number is one-third the logarithm of the number. For example the logarithm of *82500*, with the mantissa determined from a set of tables, is *4.9165*. The mantissa is the part of the logarithm to the right of the decimal point and depends only upon the sequence of numbers. The characteristic is the part of the logarithm to the left of the decimal point and is one less than the number of digits to the left of the decimal point in the original number. The logarithm of the cube root of *82500* is one-third of *4.9165*, or *1.6388*. The tables give the number which has this logarithm as *43.5*. The characteristic of the logarithm, which is the part of the logarithm to the left of the decimal point of the

logarithm, shows the location of the decimal point of the number. The number of digits to the left of the decimal point of the number is one more than the characteristic. In dividing the logarithm of the number by three, the characteristic would either divide exactly by three or there would be a remainder. The remainder would "carry-over" to the mantissa so the characteristic of the cube root would always be obtained by dividing the characteristic of the original number by three and dropping down to the next integer below. Thus, when the logarithm of the original number is *4.9165* with four as the characteristic, four divided by three gives one and one-third, which is reduced to one. The characteristic of the cube root is one. As an equation, the relation between characteristics is,

$$\frac{\text{characteristic of number}}{3} - \text{remainder} = \text{characteristic of cube root}$$

The remainder must be zero, one-third, or two-thirds. The equation can be expressed in terms of the digit count, since the characteristic is one less than the digit count. It becomes,

$$\frac{\text{digit count for number} - 1}{3} - \text{remainder} = \text{digit count for cube root} - 1$$

or,

$$\frac{\text{digit count for number}}{3} - \frac{1}{3} - \text{remainder} = \text{digit count for cube root} - 1$$

When the $-\,1$ is transposed to the left side of the equation, there results,

$$\frac{\text{digit count for number}}{3} + \frac{2}{3} - \text{remainder} = \text{digit count for cube root}$$

The digit count for the cube root must be an integer. This is the right side of the equation, and, for equality, the left side must be the same integer. The remainder is either zero, one-third or two-thirds. Hence,

$$\frac{2}{3} - \text{remainder}$$

must be, respectively, two-thirds, one-third or zero. It cannot be greater than two-thirds. Hence it is just enough to raise

$$\frac{\text{digit count for number}}{3}$$

to the next larger integer. When one-third of the digit count for the number is raised to the next larger integer, the result is the digit count for the cube root of the number.

PART B—CUBE AND CUBE ROOT WITH THE K SCALE

THE CUBE OF A NUMBER. The cube of a number is the product of the number multiplied by itself twice. The cube of a number a is equal to $a \times a \times a$. In order to find the cube of a number by use of the K scale, set the hairline of the runner to the number on the D scale and read the cube on the K scale under the hairline. There is only one place to locate a given number on the D scale. Hence, you cannot go wrong here.

> **Rule 4. (a) The Cube.** *Set the hairline of the runner to the number on the D scale. Read the cube of the number on the K scale under the hairline.*

Location of the Decimal Point. The K scale is a triple scale, containing three identical lengths. The rules for locating the decimal point in the cube of a number depend upon where the cube is located on the K scale. They are:

Case 1. *If the cube of a number is located in the left part of the K scale, the digit count for the cube is equal to two less than three times the digit count for the number.*

Case 2. *If the cube of a number is read in the center part of the K scale, the digit count for the cube is one less than three times the digit count for the number.*

Case 3. *If the cube of a number is read in the right part of the K scale, the digit count for the cube is exactly three times the digit count for the number.*

The process of finding the cube of a number is demonstrated in the following examples.

ILLUSTRATIVE EXAMPLES

25. Cube 13.5.

1) Set the hairline of the runner to *13.5* on the D scale.
2) Read the digits in the cube of *13.5* on the K scale under the hairline. The digits are *247*. Notice that it is read in the left part of the K scale, so the rule in Case 1 applies.
3) The digit count for *13.5* is two. Multiply two by three. The result is six.
4) Since the cube was read in the left part of the K scale, subtract two from six. The result is four, the digit count for the answer.
5) Then there are four digits to the left of the decimal point in the cube of *13.5*, and it is *2470*.

It is best to write down the sequence of numbers in the cube as soon as you can read it and then think about locating the decimal point.

26. Cube 0.0427.

1) Set the hairline of the runner to *0.0427* on the D scale.
2) Read the digits in the cube of *0.0427* as *78*. This is read on the K scale under the hairline. Notice that the cube is read in the center part of the K scale, so the rule in Case 2 applies.
3) The digit count for *0.0427* is minus one. Here the zero between the decimal point and the 4 is counted as a negative digit. Minus one multiplied by three is minus three.
4) Subtract one from minus three since the cube was read in the center part of the K scale; the result is minus four, the digit count for the answer.
5) Then there are minus four digits in the cube of 0.0427, which means that there must be four zeros to the right of the decimal point, between the decimal point and the 7, and the cube is 0.000078.

27. Cube 0.742.

1) Set the hairline of the runner to *0.742* on the D scale.

2) Read the digits in the cube of *0.742* on the K scale under the hairline as *41*. Notice that this is read in the right part of the K scale.
3) The digit count for *0.742* is zero. Zero multiplied by three is zero.
4) Since the cube of *0.742* was read in the right part of the K scale, there is nothing to subtract from zero.
5) There are zero digits to the left of the decimal point in the cube of *0.742* and it is *0.41*.

28. Cube 3.63.

1) Set the hairline of the runner to *3.63* on the D scale.
2) Read the digits in the cube of *3.63* on the K scale under the hairline. This is read in the center part of the K scale and is *479*.
3) The digit count for *3.63* is one. One multiplied by three is three.
4) Subtract one from three since the cube was read in the center part of the K scale. The result is two.
5) Then there are two digits to the left of the decimal point of the cube of *3.63* and it is *47.9*.

29. Cube 0.143.

1) Set the hairline of the runner to *0.143* on the D scale.
2) Read the digits in the cube of *0.143* as *293* on the K scale under the hairline. Notice that the cube is located in the left part of the K scale.
3) The digit count for *0.143* is zero. Zero multiplied by three is zero.
4) Since the cube of *0.143* was read in the left part of the K scale, subtract two from zero. The result is minus two.
5) Then there are two zeros to the right of the decimal in the cube of *0.143*, and it is *0.00293*.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

Cube each of the following numbers. Locate the decimal point in the answer.

1. 97.5	**3.** 128	**6.** 17.32	**9.** 0.203
2. 0.0866	**4.** 3.45	**7.** 39.6	**10.** 19.6
	5. 0.431	**8.** 0.915	

11. Find the volume of a cube that is 7.23 inches on a side.

12. The volume of a sphere is one-sixth times 3.14 times the cube of the diameter. Find the volume of a ball bearing that is one-fourth inch in diameter.

13. Find the volume of material required for a hollow sphere that has an outside diameter of 5.6 inches and an inside diameter of 3.9 inches.

14. Solve for x,

$$x = (9.4)^3 + 0.0341(17.2)^3$$

15. Find the cube of

$$\frac{0.707 \times 453}{66.8}$$

THE CUBE ROOT OF A NUMBER. The cube root of a number, a, is a second number, b, such that $a = b \times b \times b$. The cube root of a number is found by reversing the process of finding the cube. The hairline of the runner is set to the number on the K scale and the cube root is read on the D scale under the hairline. The K scale is a triple scale and a given number can be located in three different places on it. This raises the question of which part of the scale to use in each problem. The answer to this question depends upon the digit count for the number. When the digit count is divided by three, the result must be an integer, an integer plus one-third or an integer plus two-thirds. The part over and above an integer is called the *remainder* and the integer must be expressed so that the *remainder is positive.* The rules are:

1) When the remainder is one-third, locate the number in the left portion of the K scale.
2) When the remainder is two-thirds, locate the number in the center portion of the K scale.
3) When the remainder is zero, locate the number in the right portion of the K scale. The remainder is zero if the digit count for the number is exactly divisible by three, or if the digit count is zero.

In applying these rules to numbers such as *0.00185*, each zero between the decimal point and the first digit of the number is counted as a negative digit. Thus the digit count for *0.00185* is minus two. Minus two divided by three is minus two-thirds, but in order to have a positive remainder, this should be expressed as minus one plus one-third. The remainder is one-third. As another example consider *0.0000227*. There are four zeros between the decimal point and the first digit of the number. Hence the digit count for *0.0000227* is minus four. Minus four divided by three is minus one and one-third, but in order to have a positive remainder this would be expressed as minus two plus two-thirds. The remainder is positive and is two-thirds; it is absolutely necessary to have a positive remainder.

The numbers *1*, *10*, *100*, *1000*, etc. require special attention. When you have determined which third of the K scale to use, locate the number at the left edge of that third. For example, the number *1000* has a digit count of four. Four divided by three is one and one-third. The remainder is one-third, so the number *1000* should be located in the left third of the K scale to find its cube root. Locate it at the left edge of the left third of the K scale.

Rule 4. (b) The Cube Root. *Set the hairline of the runner to the number on the K scale. Read the cube root of the number on the D scale under the hairline.*

Location of the Decimal Point. The first step in locating the decimal point in the cube root of a number is to divide the digit count for the number by three. If the result is an integer, it is the digit count for the cube root. If the result is not an integer, then it should be increased to the next larger integer. This next larger integer is then the digit count for the cube root. Remember that to increase a negative number is to bring it closer to zero. Thus when minus two plus one-third is increased to the next larger integer, the result is minus one.

As usual the process will be illustrated with a few examples.

ILLUSTRATIVE EXAMPLES

30. Find the cube root of 1210.

1) Four is the digit count for *1210.* Four divided by three is one and one-third. The remainder is one-third. Therefore, set the hairline of the runner to *1210* in the left portion of the K scale.
2) Read the digits in the answer on the D scale under the hairline. The digits are *1066.*
3) The result of dividing the digit count for *1210* by three was one and one-third. Increase this to the next larger integer which is two.
4) Then there are two digits to the left of the decimal point in the cube root of *1210* and it is *10.66.*

31. Find the cube root of 0.0344.

1) The digit count for *0.0344* is minus one. Minus one divided by three is minus one-third, but in order to have a positive remainder, this should be expressed as minus one plus two-thirds. The remainder is two-thirds, so set the hairline of the runner to *0.0344* in the center portion of the K scale.
2) Read the digits in the cube root of *0.0344* as *325* on the D scale under the hairline.
3) The digit count for *0.0344,* divided by three was minus one plus two-thirds. The next larger integer is zero.
4) The digit count for the answer is zero, there are zero digits to the left of the decimal point in the cube root of *0.0344*, and it is 0.325.

32. Find the cube root of 468,000.

1) The digit count for *468,000* is six. Six divided by three is two. The remainder is zero, so set the hairline of the runner to *468,000* in the right portion of the K scale.
2) Read the sequence of numbers in the cube root of *468,000* as *775* on the D scale under the hairline.
3) The result of dividing the digit count for *468,000* by three was exactly two, with a remainder of zero. Two is an integer and does not have to be increased.
4) Then the digit count for the cube root of *468,000* is two and it is 77.5.

33. Find the cube root of 73,000.

1) The digit count for *73,000* is five. Five divided by three is one and two-thirds. Since the remainder is two-thirds, the hairline of the runner should be set to *73,000* in the center portion of the K scale.
2) Read the sequence of numbers in the cube root of *73,000* as *417* on the D scale under the hairline.
3) When you divided the digit count for *73,000* by three, the result was one and two-thirds. Increase this to the next larger integer which is two.
4) Then there are two digits to the left of the decimal point in the cube root of *73,000* and it is *41.7*.

34. Find the cube root of 0.0001.

1) The digit count for *0.0001* is minus three. Minus three divided by three is minus one with a remainder of zero. Therefore, set the hairline of the runner to *0.0001* in the right portion of the K scale. This must be set at the left edge of the right portion.
2) Read the digits in the cube root of *0.0001* as *464* on the D scale under the hairline.
3) The remainder, after dividing the digit count for *0.0001* by three, was zero. Hence the integer minus one stands unchanged as the digit count for the answer.
4) Then there is one zero to the right of the decimal point in the cube root of *0.0001* and it is *0.0464*.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

These problems will help you to remember what you have just learned.

Find the cube root of each of the following numbers:

1. 34.5	**3.** 3,450	**6.** 1,950,000	**9.** 0.431
2. 345	**4.** 0.00748	**7.** 5,240	**10.** 7,650
	5. 32,000	**8.** 288	

11. What is the largest cube that can be made from 30 pounds of steel? Steel weighs 0.283 pounds per cubic inch.

12. A spherical tank must be designed for a capacity of 1000 cubic feet. What must be the diameter? The volume of a sphere is one-sixth times 3.14 times the cube of the diameter.

13. Solve for x,

$$x^3 = 24.5 + (3.9)^2$$

14. Find the cube root of $\dfrac{15900}{32.2 \times 0.646}$

15. A hollow sphere is to have an external diameter of 1.25 inches and a volume of material of 0.834 cubic inches. Find the internal diameter.

BASIS OF THE PROCESS. Again we turn to logarithms and the fact that the logarithm of a product is equal to the sum of the logarithms of the numbers multiplied. The cube of a number is equal to the number multiplied by itself twice, so the logarithm of the cube is equal to the logarithm of the number plus the logarithm of the number plus the logarithm of the number. Or, the logarithm of the cube is equal to three times the logarithm of the number. The logarithm of *11.3* is *1.0531.* The mantissa, which is the part of the logarithm to the right of the decimal point depends only upon the sequence of numbers in the number *11.3.* The mantissa can be found from a set of tables of logarithms. The characteristic, which is the part of the logarithm to the left of the decimal point, depends only on the location of the decimal point in the original number. It is always one less than the digit count for the number. The cube of *11.3* can be found by means of logarithms. Thus,

$$\text{Logarithm of } 11.3 = 1.0531$$
$$3 \times \text{logarithm of } 11.3 = 3.1593$$

The logarithm of the cube of *11.3* is *3.1593.* A table of logarithms gives the sequence of numbers in the cube of *11.3* as *1443.* The characteristic is *3* so there are four digits to the left of the decimal point. Hence, the cube of *11.3* is *1443.*

On the slide rule this would be found by setting the hairline of the runner to *11.3* on the D scale. The distance from the left end of the D scale to *11.3* represents the mantissa of the logarithm of 11.3. The cube of *11.3* is read on the K scale under the hairline, with the distance from the left end of the K scale to the

cube representing the mantissa of the cube. The K scale is a triple scale and a given range of numbers occupies one-third as much space on it as on the D scale. Conversely, with equal distances on the K scale and D scale, the distance on the K scale represents a mantissa three times as large as the distance on the D scale. Hence, the mantissa of the number on the K scale that is under the hairline is three times as large as the mantissa of the number on the D scale that is under the hairline. Therefore the number on the K scale is the cube of the number on the D scale, or the number on the D scale is the cube root of the number on the K scale.

Location of the Decimal Point by Logarithms. If the mantissa of the logarithm of the number that is to be cubed is less than *0.333,* there is no "carry-over" to the characteristic when multiplying by three. Hence the characteristic of the cube is exactly three times the characteristic of the number. If the mantissa of the number is less than *0.333,* the number will be located in the left one-third of the D scale and the cube will be read in the left portion of the K scale. As an equation the relation between characteristics in this case is,

$$3 \times \text{characteristic of number} = \text{characteristic of cube}$$

When this is expressed in terms of the digit count, it becomes,

$$3\ (\text{digit count for number} - 1) = \text{digit count for cube} - 1$$

or,

$$3 \times \text{digit count for number} - 3 = \text{digit count for cube} - 1$$

When the -1 is transposed to the left side of the equation, there results,

$$3 \times \text{digit count for number} - 2 = \text{digit count for cube}$$

In words this is: the digit count for the cube of a number is two less than three times the digit count for the original number. But this is true only when the cube is read in the left portion of the K scale. The relation between digits can be rewritten to show when to locate a number in the left portion of the K scale for the purpose of finding its cube root. After transposing the (-2) the equation is,

$$3 \times \text{digit count for number} = \text{digit count for cube} + 2$$

When the equation is divided by three, it becomes,

$$\text{digit count for number} = \frac{\text{digit count for cube}}{3} + \frac{2}{3}$$

or, rewording for use in finding the cube root of a number,

$$\text{digit count for cube root} = \frac{\text{digit count for number}}{3} + \frac{2}{3}$$

since the cube root of a number bears the same relation to the number as a given number bears to its cube. The digit count for the cube root must be an integer. Therefore, the right side of the equation must be an integer and if the two-thirds is exactly enough to increase

$$\frac{\text{digit count for number}}{3}$$

to the next larger integer, then,

$$\frac{\text{digit count for number}}{3}$$

must be an integer plus one-third. This is the basis for the rule that the digit count for the cube root is obtained by dividing the digit count for the original number by three and increasing the result to the next larger integer. This applies to all cases in which the number is located in the left portion of the K scale. After dividing the digit count by three, the remainder is one-third. Hence, whenever the remainder is one-third, the number should be located in the left portion of the K scale and its cube root should be read directly above it on the D scale.

If the mantissa of the logarithm of the number that is to be cubed is between *0.333* and *0.667*, the number is located in the middle one-third of the D scale, and the cube is read in the center portion of the K scale. Also, when the mantissa is multiplied by three, there is a carry-over of one to the characteristic. Hence, the characteristic of the cube of the number is one more than three times the characteristic of the original number. As an equation, this is,

$$3 \times \text{characteristic of number} + 1 = \text{characteristic of cube}$$

Changing to digits,

3 (digit count for number $-$ 1) $+$ 1 $=$ digit count for cube $-$ 1
or,

$3 \times$ digit count for number $- 3 + 1 =$ digit count for cube $-$ 1
simplifying,

$$3 \times \text{digit count for number} - 1 = \text{digit count for cube}$$

Thus, when the cube is read in the center portion of the K scale the digit count for the cube is one less than three times the digit count for the original number. The equation can be rewritten as,

$$3 \times \text{digit count for number} = \text{digit count for cube} + 1$$

or, for use in finding the cube root of a number,

$$3 \times \text{digit count for cube root} = \text{digit count for number} + 1$$

After dividing by three, this becomes,

$$\text{digit count for cube root} = \frac{\text{digit count for number}}{3} + \frac{1}{3}$$

Each side of this equation must be an integer. If the right side is an integer, then

$$\frac{\text{digit count for number}}{3}$$

must be an integer plus two-thirds. From this follows the rule that if the remainder is two-thirds when the digit count is divided by three, the number is to be located in the middle portion of the K scale for the purpose of finding the cube root. Also it is apparent that the digit count for the cube root is obtained by dividing the digit count for the original number by three and increasing this to the next larger integer.

If the mantissa of the logarithm of the number is between *0.667* and *1.000,* the number is located in the right one-third of the D scale and its cube is read in the right portion of the K scale. Also, when the logarithm is multiplied by three, there is a carry-over of two from the mantissa to the characteristic of the cube. Thus the characteristic of the cube of a number is two more than three times the characteristic of the original number, when the cube is read in the right portion of the K scale. As an equation, the characteristic relation is,

$$3 \times \text{characteristic of number} + 2 = \text{characteristic of cube}$$

When this is expressed in terms of digit counts, it is,

3 (digit count for number − 1) + 2 = digit count for cube − 1

or,

3 × digit count for number − 3 + 2 = digit count for cube − 1

The integers cancel, leaving,

$$3 \times \text{digit count for number} = \text{digit count for cube}$$

Thus, when the cube is read in the right portion of the K scale, the digit count for the cube is exactly three times the digit count for the original number. The equation can be rewritten as,

$$\text{digit count for number} = \frac{\text{digit count for cube}}{3}$$

or, rewording for use in finding the cube root of a number,

$$\text{digit count for cube root} = \frac{\text{digit count for number}}{3}$$

When the digit count for a number is exactly divisible by three, the number should be located in the right portion of the K scale for the purpose of finding its cube root. Also, the digit count for the cube root of the number is exactly one-third of the digit count for the number.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

Find the cube of each of the following numbers:

1. 3.93
2. 12.5
3. 0.71
4. 21.4
5. 0.532
6. 7.6
7. 4.7
8. 19.5
9. 1.76
10. 28.7

Find the cube root of each of the following numbers:

11. 35
12. 0.67
13. 152
14. 9670
15. 2.73
16. 424
17. 87.5
18. 0.055
19. 1.78
20. 15,700

OTHER TYPES OF SLIDE RULES. The marking is usually the same for the K scale, no matter what the type of slide rule. However, the location of the K scale is not the same on all slide rules. With some, this scale is on the lower part of the stock as shown in Fig. 33. Here the mark on the lower edge of the runner locates a number on the K scale. The cube root of this

number is read on the D scale under the hairline of the runner. In Fig. 33, the number *23.5* is located in the center third of the K scale by means of the mark on the runner. The cube root of *23.5* is *2.87* and so *2.87* on the D scale is under the hairline of the runner.

Occasionally the K scale is located on the front of the stock, near the upper edge. In such a case, the hairline of the runner locates at one time a number on the K scale and also a number on the D scale. When the K scale is in such a location, it can be used in the manner described in this chapter.

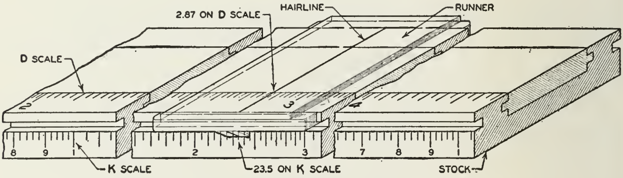


Fig. 33

On some types of slide rules, this scale is designated by another letter instead of K. This makes no difference in the use of the scale.

REVIEW PROBLEMS

Answers to Review Problems are not given in the back of the book. Readers who are working alone may check their answers by working the problems longhand.

Find the cube of each of the following numbers:

1. 9.87
2. 16.7
3. 0.0227
4. 0.473
5. 6.93
6. 1184
7. 1.184
8. 37.2
9. 5.84
10. 0.00919

Find the cube root of each of the following numbers:

11. 29,600,000
12. 37.3
13. 1672
14. 0.00531
15. 0.373
16. 4.52
17. 56.8
18. 0.0000785
19. 0.0638
20. 19.5
21. 2.84
22. 0.00177
23. 115,600
24. 758
25. 483

For practice, work problems in Part A by this method.

CHAPTER IX

SINES AND COSINES

Many engineering and shop problems require the use of the sine or cosine of an angle. The quickest way to find one of these functions is to get it from the slide rule. You will find it very convenient to be able to do this and the value you get is precise enough for most practical calculations.

THE SINE OF AN ANGLE. The problem is to find the sine of an angle. The angle is expressed in degrees and minutes, and the sine is always a decimal fraction, such as *0.749* or *0.0285.* The first step in finding the sine of an angle with the slide rule is to pull the slide to the right. Then turn the rule over so that you can see the back of it. When you have done this, you can see a portion of the sine scale on the slide, identified by the letter *S* at the right-hand end. Next adjust the slide so that the angle for which you want the sine is under the mark* on the celluloid insert at the right-hand end of the rule. Then turn the slide rule over so that you again see the front of it. The sine of the angle is located on the B scale under the right index of the A scale. The B scale is a double scale and for the purpose of finding the sine of an angle, the numbers on it should be considered as ranging from *0.01* at the left index to *1* at the right index. The center index has the value *0.1* as the sine of an angle. Numbers in the left half are between *0.01* and *0.1*. Numbers in the right half are between *0.1* and *1*. For instance, the number *4* in the left half of the B scale would be read as *0.04* as the sine of an angle; the number *4* in the right half of the B scale would be read as *0.4*.

*Some slide rules have no mark on the celluloid insert. In such a case, use the right edge of the celluloid to mark the angle. Some other slide rules have no celluloid insert at all but only a mark on the wood; this mark can be used to locate the angle on the sine scale.

Rule 5. (a) The Sine. *Set the angle on the sine scale under the mark on the celluloid insert. Read the sine of the angle on the B scale under the right index of the A scale.*

ILLUSTRATIVE EXAMPLES

The following examples illustrate the method. Follow each step on your own slide rule.

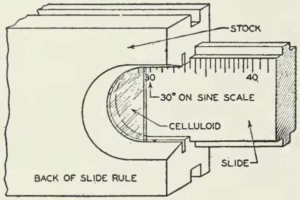


Fig. 34

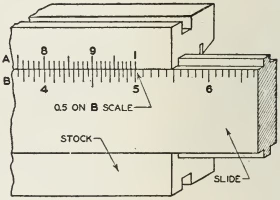


Fig. 35

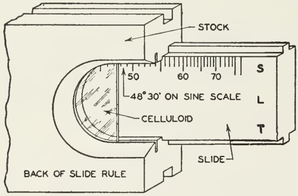


Fig. 36

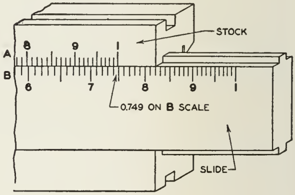


Fig. 37

1. Find the sine of 30°.

1) Pull the slide to the right and turn the slide rule over.
2) Adjust the slide so that *30°* on the sine scale is under the mark on the celluloid. Fig. 34 shows the position of the slide for this setting.
3) Turn the slide over again and read the sine of *30°* as *0.5* on the B scale under the right index of the A scale. Fig. 35 is a view of the front of the slide for this reading. You know that the answer is *0.5* and not *0.05* because it is read in the right half of the B scale and hence must be between *0.1* and *1.*

2. Find the sine of 48°30′.

1) Pull the slide to the right and turn the rule over.
2) Adjust the slide so that *48°30′* on the sine scale is under the mark on the celluloid. Remember that *30′* is one-half of 1° and so *48°30′* is halfway between *48°* and *49°*. Fig. 36 shows this.
3) Turn the slide rule over again and read the sine of *48°30′* as *0.749* on the B scale under the right index of the A scale. Fig 37 shows the right ends of the A and B scales for this setting.

3. Find the sine of 3°25′.

1) Pull the slide to the right and turn the rule over.

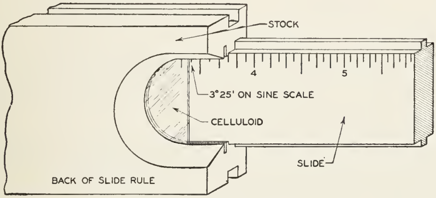


Fig. 38

2) Adjust the slide so that *3°25′* on the sine scale is under the mark on the celluloid. Fig. 38 shows this setting. Notice that each space between *3°* and *4°* on the sine scale represents 5′ of angle.
3) Turn the slide rule over again and read the sine of *3°25′* on the B scale under the right index of the A scale. The digits in the sine are read as *596*. Since you read it in the left half of the B scale it must be between *0.01* and *0.1*. Therefore, it is *0.0596*.

4. Find the sine of 1°38′.

1) Pull the slide to the right and turn the slide rule over.
2) Adjust the slide so that *1°38′* on the sine scale is under the mark on the celluloid.
3) Turn the rule over again and read the sine of *1°38′* as *0.0285* on the B scale under the right index of the A scale.

Remember that there are 60′ in 1°. Consequently, the distance between two consecutive degree marks on the sine scale represents 60′. If the marking of the sine scale seems confusing, go back to the chapter on "Scales of the Slide Rule" and read the pages devoted to the sine scale.

PRACTICE PROBLEMS

After you have worked all the following problems, check your answers with the correct answers shown in the back of the book.

Find the sine of each of the following angles:

1. 67°	**3.** 10°30′	**6.** 4°28′	**9.** 2°45′
2. 53°10′	**4.** 60°	**7.** 26°32′	**10.** 73°
	5. 45°	**8.** 15°20′	

11. The vertical component of a force is equal to the product of the force and the sine of the angle which it makes with the horizontal. Find the vertical component of a force of 230 pounds at an angle of 40° with the horizontal.

12. A force at an angle of 22°30′ with the horizontal has a vertical component of 18 pounds. Find the force.

Verify each of the following equations:

13. $\sin 65° = 2 \sin 32°30' \sin 57°30'$

14. $\sin 50° = 2 \sin 25° \sin 65°$

15. $\sin 55° + \sin 10° = 2 \sin 32°30' \sin 67°30'$

16. $\sin^2 40° + \sin^2 50° = 1$

17. $\sin^2 30° = \frac{1}{2} \sin 30°$

18. $\dfrac{7.4}{\sin 25°} = \dfrac{12.36}{\sin 45°}$

Basis of the Process. The B scale can be thought of as a scale, ranging from *0.01* to *1,* in which the distance from the left index to the number represents the logarithm of the number. The sine scale, which is of the same length as the B scale, is laid off to correspond with the B scale so that the distance from the left end represents the logarithm of the sine of the angle. Thus the relation between the number on the B scale and the angle on the sine scale is,

logarithm of number on B scale = logarithm of sine of angle on sine scale

If the logarithms of two quantities are equal, the two quantities must be equal. Thus,

number on B scale = sine of angle on sine scale

THE SINE OF A VERY SMALL ANGLE. The smallest angle on the sine scale is *0°34′*. Hence the sine of an angle smaller than this cannot be found in the usual way. Such an angle would be expressed in minutes and seconds as, for instance, *20′33″*.* The first step in finding the sine is to convert the number of seconds to a decimal fraction of a minute. This is done by dividing by 60 since there are 60″ in 1′. For example, *33* divided by *60* is *0.55*, so *20′33″* is *20.55′*. Next the number of minutes, for instance, *20.55*, is divided by *3440*. The result is the sine of the angle as a decimal fraction. The sine of the angle *20′33″* is *0.00598*.

ILLUSTRATIVE EXAMPLES

5. Find the sine of 15′45″.

1) Convert *45″* to a decimal fraction of *1′* by dividing *45* by *60*. (Do this by setting the hairline of the runner to *45* on the D scale. Next bring *60* on the C scale under the hairline. Read the answer on the D scale under the right index of the C scale.) The result is *0.75*.

2) Thus *15′45″* is *15.75′*.

3) Divide *15.75* by *3440*. (The first step in this division is to set the hairline of the runner to *15.75* on the D scale. Next slide *3440* on the C scale under the hairline. Read the digits in the result as *458* on the D scale under the right index of the C scale. The digit count† for *15.75* is two, and four is the digit count for *3440*. In accordance with the rules given in the chapter on Division, in this case subtract four from two. The result is minus two and this is the digit count for the answer. This means that there must be two zeros immediately following the decimal point.) The result is *0.00458*. Thus, the sine of *15′45″* is *0.00458*.

---

*The double mark ″ is read as seconds.

†Digit counts are explained on p. 50.

6. Find the sine of 0′23″.

1) Convert *23″* to a decimal fraction of *1′* by dividing *23* by *60*. The result is *0.383*.
2) Thus *0′23″* is *0.383′*.
3) Divide *0.383* by *3440*.
4) The result is *0.0001112* and this is the sine of *0′23″*.

7. Find the sine of 6′0″.

1) This is *6.0′*.
2) Divide *6* by *3440*.
3) The result is *0.001745*. This is the sine of *6′0″*.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers at the back of the book.

Find the sine of each of the following angles:

1. 32′20″	**3.** 1′12″	**6.** 14′05″	**9.** 0′37″
2. 25′30″	**4.** 11′0″	**7.** 8′15″	**10.** 21′17″
	5. 19′52″	**8.** 6′27″	

11. Find the vertical component of a force of 2100 pounds at an angle of 16′30″ with the horizontal.

12. A train travels for 6.5 miles up a grade which makes an angle of 45′ with the horizontal. What is the change in altitude of the train? (*Hint:* Multiply 6.5 miles by the sine of 45′.)

13. How far must an automobile travel on a 30′ grade to gain 100 feet in altitude?

14. The hypotenuse of a certain right triangle is 28.3 inches in length and makes an angle of 25′ with the base. Find the altitude of the triangle. (*Hint:* Multiply 28.3 by the sine of 25′.)

Basis of the Process. The sine of a very small angle is very nearly equal to the angle expressed in radians.* The difference between the sine and the value of the angle in radians can be neglected. Thus the problem is to express the angle in radians. One radian is 57.3 degrees. If the angle, expressed in minutes, is divided by 60, the result is the angle expressed in degrees. Then

*The radian is a unit by means of which an angle may be expressed or measured. It can be shown by means of calculus that the sine of a very small angle is approximately equal to the angle in radians.

if the angle in degrees is divided by 57.3, the result is the angle in radians, and this value can be used as the sine of the angle. The process is,

$$\frac{\text{angle in minutes}}{60 \times 57.3} = \text{angle in radians} = \text{sine of angle}$$

or,

$$\frac{\text{angle in minutes}}{3440} = \text{sine of angle}$$

This expression can be used when the angle is less than about 6°. It is only necessary to use it when the angle is less than 0°34′.

THE COSINE OF AN ANGLE. There is no cosine scale on the slide rule but none is necessary. The cosine of an angle can be found by using the sine scale, since the cosine of an angle is equal to the sine of 90° minus the angle. Thus the cosine of 30° is equal to the sine of 90° − 30°, that is, the sine of 60°. The sine of 60° can be found in the usual way.

ILLUSTRATIVE EXAMPLES

8. Find the cosine of 50°.

1) The cosine of *50°* is equal to the sine of *90° − 50°*, or the sine of *40°*.
2) The sine of *40°* is *0.643*. Hence the cosine of *50°* is *0.643*.

There are only 60′ in 1°. Remember this when you subtract an angle from 90°.

9. Find the cosine of 63°20′.

1) The cosine of *63°20′* is equal to the sine of *90° − 63°20′*, or the sine of *26°40′*.
2) The sine of *26°40′* is *0.449*. Hence, *0.449* is the cosine of *63°20′*.

10. Find the cosine of 42°15′.

1) The cosine of *42°15′* is equal to the sine of *90° − 42°15′*, which is the sine of *47°45′*.
2) The sine of *47°45′* is *0.740*, so the cosine of *42°15′* is *0.740*.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers at the back of the book.

Find the cosine of each of the following angles:

1. $34°15'$	3. $51°40'$	6. $88°30'$	9. $76°30'$
2. $25°30'$	4. $47°30'$	7. $89°45'$	10. $65°40'$
	5. $20°$	8. $59°10'$	

THE SINE OF AN ANGLE BETWEEN 70° AND 90°. The usual method of determining the sine of an angle is not very precise if the angle is between 70° and 90°, because the distance between 70° and 90° on the sine scale is so small. However, the sine of any such angle θ can be found by using the formula,

$$\sin\theta = \sqrt{1 - \sin^2 (90° - \theta)}$$

If the angle θ is larger than 70°, the sine of $90° - \theta$ can be found easily from the slide rule. You already know how to square a number and how to find the square root. (If you do not, read the chapter on "Square and Square Root.") Thus you can work out the formula. With practice you will be able to do it without writing down any intermediate results.

ILLUSTRATIVE EXAMPLES

11. Find the sine of 85°.

1) $90° - 85° = 5°$. The sine of *5°* is *0.0872*.
2) Square *0.0872*. The result is *0.0076*. This is $\sin^2 (90° - \theta)$.
3) Next find $1 - \sin^2 (90° - \theta)$. This is $1 - 0.0076 = 0.9924$.
4) The square root of *0.9924* is *0.996*. Hence the sine of *85°* is *0.996*.

12. Find the sine of 77°30′.

1) $90° - 77°30' = 12°30'$. The sine of *12°30′* is *0.216*.
2) Square *0.216*. The result is *0.0467*.
3) $1 - 0.0467 = 0.9533$.
4) Find the square root of *0.9533*. This is *0.977*, so the sine of *77°30′* is *0.977*.

13. Find the sine of 81°15′.

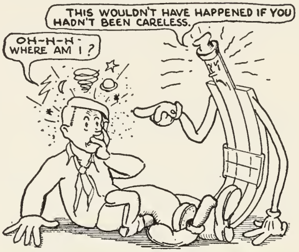
1) $90° - 81°15' = 8°45'$. The sine of *8°45′* is *0.152*.
2) Square *0.152*. The result is *0.0232*.
3) $1 - 0.0232 = 0.9768$.
4) Find the square root of *0.9768*. The result is *0.988*, so *0.988* is the sine of *81°15′*.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers at the back of the book.

Find the sine of each of the following angles:

1. 82°30′
2. 71°45′
3. 76°10′
4. 81°50′
5. 83°40′
6. 73°15′
7. 79°20′
8. 78°30′
9. 80°
10. 75°22′



YOU'LL GET TANGLED UP IF YOU'RE CARELESS

Basis of the Process. The basis of this process is the trigonometric identity,

$$\sin^2 \theta + \cos^2 \theta = 1$$

This is true for any angle θ. The term $\cos^2 \theta$ can be transposed, leaving,

$$\sin^2 \theta = 1 - \cos^2 \theta$$

Then substitute for $\cos \theta$,

$$\cos \theta = \sin (90° - \theta)$$

This gives,

$$\sin^2 \theta = 1 - \sin^2 (90° - \theta)$$

Next take the square root of each side. The result is,

$$\sin \theta = \sqrt{1 - \sin^2 (90° - \theta)}$$

THE SINE OF AN ANGLE GREATER THAN 90°. The sine of an angle greater than 90° cannot be found directly from the slide rule. However, the sine of such an angle can always be expressed in terms of the sine of an angle between 0° and 90°, and the sine of an angle between 0° and 90° can be found directly from the slide rule.

Angles between 90° and 180°. For any angle θ between 90° and 180°, the following equation is true,

$$\sin \theta = \sin (180° - \theta)$$

If θ is between 90° and 180°, then $(180° - \theta)$ must be between 0° and 90°. The sine of $(180° - \theta)$ can be found with the slide rule.

14. Find the sine of 145°.

1) Sin *145°* = sin (*180°* − *145°*) = sin *35°*.
2) The sine of *35°* is *0.570*. Hence, the sine of *145°* is *0.570*.

15. Find the sine of 166°40′.

1) Sin *166°40′* = sin (*180°* − *166°40′*) = sin *13°20′*.
2) The sine of *13°20′* is *0.231*, so the sine of *166°40′* is *0.231*.

Angles between 180° and 270°. When an angle θ is between 180° and 270°, you can use the relation,

$$\sin \theta = -\sin (\theta - 180°)$$

If θ is between 180° and 270°, then $(\theta - 180°)$ is between 0° and 90°. Hence you can find the sine of $(\theta - 180°)$ directly from the slide rule.

16. Find the sine of 220°.

1) Sin *220°* = − sin (*220°* − *180°*) = − sin *40°*.
2) The sine of *40°* is *0.643*. Thus the sine of *220°* is (− *0.643*).

17. Find the sine of 243°30′.

1) Sin *243°30′* = − sin (*243°30′* − *180°*) = − sin *63°30′*.
2) The sine of *63°30′* is *0.895*. Hence, the sine of *243°30′* is (− *0.895*).

Angles between 270° and 360°. The equation that is to be used when the angle θ is between 270° and 360° is,

$$\sin \theta = -\sin (360° - \theta)$$

When θ is between 270° and 360°, the angle $(360° - \theta)$ is between 0° and 90°.

18. Find the sine of 293°20′.

1) Sin *293°20′* = − sin (*360°* − *293°20′*) = − sin *66°40′*.
2) The sine of *66°40′* is *0.918*. Therefore, the sine of *293°20′* is (− *0.918*).

19. Find the sine of 337°45′.

1) Sin *337°45′* = − sin (*360°* − *337°45′*) = − sin *22°15′*.
2) The sine of *22°15′* is *0.379*, so the sine of *337°45′* is (−*0.379*).

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers at the back of the book.

Find the sine of each of the following angles:

1. 205°	**3.** 170°	**6.** 330°	**9.** 321°20′
2. 310°	**4.** 123°15′	**7.** 188°45′	**10.** 212°50′
	5. 242°30′	**8.** 171°15′	

THE COSINE OF AN ANGLE GREATER THAN 90°. The cosine of any large angle can be expressed in terms of the sine of an angle between 0° and 90°. Since you can find the sine of an angle between 0° and 90° with the slide rule, you can find the cosine of any large angle.

Angles between 90° and 180°. When the angle θ is between 90° and 180°, use the equation,

$$\cos \theta = -\sin (\theta - 90°)$$

If θ is between 90° and 180°, then $(\theta - 90°)$ is between 0° and 90°.

20. Find the cosine of 135°.

1) Cos *135°* = – sin (*135°* – *90°*) = – sin *45°*.
2) The sine of *45°* is *0.707*. Hence, the cosine of *135°* is (– *0.707*).

21. Find the cosine of 98°10′.

1) Cos *98°10′* = – sin (*98°10′* – *90°*) = – sin *8°10′*.
2) The sine of *8°10′* is *0.142*. Therefore, the cosine of *98°10′* is (– *0.142*).

Angles between 180° and 270°. For any angle θ between 180° and 270°, the following equation is true,

$$\cos \theta = -\sin (270° - \theta)$$

When θ is between 180° and 270°, the angle $(270° - \theta)$ is between 0° and 90°.

22. Find the cosine of 243°30′.

1) Cos *243°30′* = – sin (*270°* – *243°30′*) = – sin *26°30′*.
2) The sine of *26°30′* is *0.446*, so the cosine of *243°30′* is (– *0.446*).

23. Find the cosine of $214°40'$.

1) Cos $214°40' = -\sin(270° - 214°40') = -\sin 55°20'$.
2) The sine of $55°20'$ is *0.821*. Hence, the cosine of $214°40'$ is (-0.821).

Angles between 270° and 360°. If the angle θ is between 270° and 360°, you can use,

$$\cos\theta = \sin(\theta - 270°)$$

The angle $(\theta - 270°)$ must be between 0° and 90° if θ is between 270° and 360°.

24. Find the cosine of $288°45'$.

1) Cos $288°45' = \sin(288°45' - 270°) = \sin 18°45'$.
2) The sine of $18°45'$ is *0.321*, so the cosine of $288°45'$ is *0.321*.

25. Find the cosine of $312°10'$.

1) Cos $312°10' = \sin(312°10' - 270°) = \sin 42°10'$.
2) The sine of $42°10'$ is *0.671*. Hence, the cosine of $312°10'$ is *0.671*.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers at the back of the book.

Find the cosine of each of the following angles:

1. 150°	**3.** 281°15′	**6.** 135°	**9.** 315°
2. 117°30′	**4.** 330°	**7.** 222°30′	**10.** 157°30′
	5. 260°10′	**8.** 95°40′	

THE ARC SINE OF A NUMBER. You will find many problems in which you calculate the sine of an angle by division or multiplication, and then want to know what the angle is. The sine of an angle is a number. The arc sine of a number is the angle which has that number for its sine. The process of finding the angle when its sine is known is just the reverse of the process of finding the sine when the angle is known. You adjust the slide so that the sine of the angle on the B scale is under the right index of the A scale. Then you turn the slide rule over and read the angle, in degrees and minutes, on the sine scale under the mark on the celluloid insert.

The following examples will make the process clear. Carry them out on your own slide rule. Remember that when working with the sine of an angle, the B scale is thought of as extending from *0.01* at the left index to *1* at the right index. A number between *0.01* and *0.1* is located in the left half of the B scale, and a number between *0.1* and *1* is located in the right half.

> **Rule 5. (b) The Arc Sine.** *Set the number on the B scale under the right index of the A scale. Read the angle which is the arc sine of the number on the sine scale under the mark on the celluloid insert.*

ILLUSTRATIVE EXAMPLES

26. Find arc sine 0.346; that is, find the angle whose sine is *0.346*.

1) Adjust the slide so that *0.346* in the right half of the B scale is under the right index of the A scale.
2) Turn the slide rule over and read the angle whose sine is *0.346* on the sine scale under the mark on the celluloid insert. This angle is *20°15′*.

27. Find arc sine 0.0346.

1) Adjust the slide so that *0.0346* in the left half of the B scale is under the right index of the A scale.
2) Turn the slide rule over and read the angle whose sine is *0.0346* on the sine scale under the mark on the celluloid. This angle is *1°59′*.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers in the back of the book.

Find the arc sine for each of the following numbers:

1. 0.748	**3.** 0.0569	**6.** 0.205	**9.** 0.350
2. 0.157	**4.** 0.667	**7.** 0.0835	**10.** 0.500
	5. 0.430	**8.** 0.905	

THE ARC SINE OF A NUMBER LESS THAN 0.01. The arc sine of a number less than 0.01 cannot be found in the

usual way, since the B scale is considered to extend from 0.01 to 1 in working with the sine or arc sine. However, for any number less than 0.01, the arc sine, expressed in minutes, is equal to 3440 times the number. Thus arc sine *0.005* is *17.2′*. Here the angle contains a decimal fraction. This decimal fraction can be converted to seconds of angle by multiplying by 60 since there are 60″ in 1′. The decimal fraction *0.2* is then, $0.2 \times 60 = 12''$, so the angle is *17′12″*.

ILLUSTRATIVE EXAMPLES

28. Find arc sine 0.00625.

1) Multiply *3440* by *0.00625*. The result is *21.5*. Hence, the angle is *21.5′*.
2) Multiply the decimal fraction, *0.5,* by *60*. The result is *30,* and this is the number of seconds. Hence arc sine *0.00625* is *21′30″*.

29. Find arc sine 0.0001.

1) Multiply *3440* by *0.0001*. The result is *0.344,* so the angle is *0.344′*.
2) Convert this decimal fraction to seconds of angle by multiplying by *60*. The result is *20.7*. Therefore, arc sine *0.0001* is *20.7″*. To the nearest second, this is *21″*.

PRACTICE PROBLEMS

After you have worked all of the problems, check your answers with the correct answers in the back of the book.

Find the arc sine of each of the following numbers. Express the result in minutes and seconds:

1. 0.0098	**3.** 0.000725	**6.** 0.000875	**9.** 0.00684
2. 0.00331	**4.** 0.001932	**7.** 0.00386	**10.** 0.001
	5. 0.00541	**8.** 0.001203	

THE ARC COSINE OF A NUMBER. The arc cosine of a number is the angle which has that number for its cosine. You cannot find the arc cosine directly. However, you can find the arc sine, that is, the angle which has the number for its sine. Then you use the fact that the cosine of an angle is the sine of 90° minus the angle. Or, vice versa, the cosine of 90° minus the angle is equal to the sine of the angle. If this statement is made in terms of the arc cosine and arc sine, it is: The arc cosine of a number is

$90°$ minus the arc sine of the number. You know how to find the arc sine. After you have it as an angle, subtract it from $90°$. The result is the arc cosine.

ILLUSTRATIVE EXAMPLES

30. Find arc cosine 0.55.

1) First find the arc sine of *0.55*. This is *33°20′*.
2) The arc cosine of *0.55* is $90° - 33°20'$, or *56°40′*.

31. Find arc cosine 0.093.

1) Find the arc sine of *0.093*. This is *5°20′*.
2) The arc cosine of *0.093* is $90° - 5°20'$, or *84°40′*.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

Find the arc cosine of each of the following numbers:

1. 0.852
2. 0.304
3. 0.278
4. 0.187
5. 0.500
6. 0.712
7. 0.667
8. 0.100
9. 0.932
10. 0.417

THE ARC SINE OF A NUMBER BETWEEN 0.95 AND 1. The usual method of finding the arc sine of a number lacks precision when the number is between 0.95 and 1. There is, however, a formula which will give good results. If the number be designated by x, and the arc sine of the number by θ, then,

$$\theta = 90° - \text{arc sine } \sqrt{1 - x^2}$$

The problem is to find the angle θ which has x for its sine. Start by calculating $\sqrt{1 - x^2}$. Then find arc sine $\sqrt{1 - x^2}$ in the usual way. Last, subtract arc sine $\sqrt{1 - x^2}$ from *90°*. The result is θ which is arc sine x. Try it in these examples and problems. The method is very useful and is worth learning. With practice, you can learn to work without writing down any intermediate results.

ILLUSTRATIVE EXAMPLES

32. Find the arc sine of 0.98.

1) The value of x is *0.98*. Square *0.98*. This is *0.96*.
2) Subtract *0.96* from *1*. The result is *0.04*.

3) Find the square root of *0.04*. This is *0.2*.
4) The angle $(90^\circ - \theta)$ is arc sine *0.2*. This is *11°32′*.
5) The angle θ is $90^\circ - 11^\circ 32'$, or *78°28′*. This is arc sine of *0.98*. Try to get it as close in the usual way.

33. Find the arc sine of 0.957.

1) The value of x is *0.957*. Square it. You obtain *0.916*.
2) Subtract *0.916* from *1*. The result is *0.084*.
3) The square root of *0.084* is *0.290*.
4) The angle $(90^\circ - \theta)$ is arc sine *0.290* and is *16°50′*.
5) The angle θ is $90^\circ - 16^\circ 50'$, or *73°10′*. This is arc sine of *0.957*.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

Find the arc sine of each of the following numbers:

1. 0.961	**3.** 0.984	**6.** 0.968	**9.** 0.995
2. 0.977	**4.** 0.990	**7.** 0.970	**10.** 0.965
	5. 0.950	**8.** 0.972	

Basis of the Process. The process rests on the trigonometric identity,

$$\sin^2 \theta + \cos^2 \theta = 1$$

which is true for any angle θ. You know *sin* θ, which we will call x and you want to find θ. Replacing *sin* θ by x, the equation becomes,

$$x^2 + cos^2 \theta = 1$$

When the term x^2 is transposed to the right side of the equation, there results,

$$\cos^2 \theta = 1 - x^2$$

Next take the square root of each side. This leaves,

$$\cos \theta = \sqrt{1 - x^2}$$

But,

$$\cos \theta = \sin (90^\circ - \theta)$$

When this is substituted, the equation becomes,

$$\sin (90^\circ - \theta) = \sqrt{1 - x^2}$$

This can be rewritten as,

$$90^\circ - \theta = \text{arc sine } \sqrt{1 - x^2}$$

from the definitions of the sine and arc sine. A last rewriting gives

$$\theta = 90^\circ - \text{arc sine } \sqrt{1 - x^2}$$

which is the desired result.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

Find the arc cosine of each of the following numbers:

1. 0.622 **2.** 0.183 **3.** 0.0795 **4.** 0.928 **5.** 0.513 **6.** 0.800 **7.** 0.047 **8.** 0.250 **9.** 0.333 **10.** 0.985

Find the arc sine of each of the following numbers:

11. 0.125 **12.** 0.900 **13.** 0.625 **14.** 0.342 **15.** 0.0660

Find the sine of each of the following angles:

16. 152°30′ **17.** 324°15′ **18.** 248°45′ **19.** 351°20′ **20.** 105°

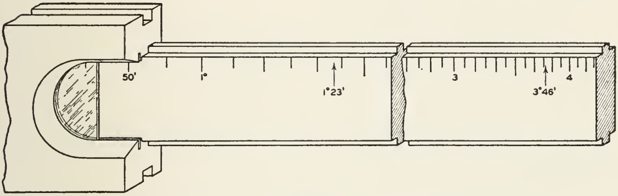


Fig. 39

Find the cosine of each of the following angles:

21. 2°28′ **22.** 137°30′ **23.** 88°30′ **24.** 50°17′ **25.** 254°40′ **26.** 70°10′ **27.** 176°32′ **28.** 292°52′ **29.** 98°50′ **30.** 89°38′

OTHER TYPES OF SLIDE RULES. Mannheim slide rules have the sine scale located the same as on the slide rule which is used for this book; that is, it is on the back of the slide. The only difference is in the marking of the sine scale. An example of different marking is shown in Fig. 39, in which appears the left portion of a sine scale. Here each space represents 5 minutes of angle. The angles *1°23′* and *3°46′* are shown in their correct locations. With the exception of the marking of the scale, the instruction material in this chapter applies to any Mannheim slide rule.

REVIEW PROBLEMS

Answers to Review Problems are not given in the back of the book. Readers who are working alone may check their answers if desired by looking up the sine or cosine in a set of tables of trigonometric functions.

By this time you know how to find the sine or cosine of any angle. Also you know how to find the arc sine and arc cosine. Practice on these problems and questions. You can learn a process very quickly, but unless you use it you will forget it quickly.

1. Find the cosine of arc sine 0.600.

2. Find the sine of arc cosine 0.45.

3. The base of a right triangle is equal to the hypotenuse times the cosine of the angle between the hypotenuse and the base. Find the base of a right triangle of which the hypotenuse is 11.27 inches at an angle of 35° with the base.

4. Find the altitude of the triangle described in problem 3.

5. What is the angle between the base and hypotenuse of a right triangle of which the base is 126 feet and the hypotenuse is 234 feet?

6. Find the angle between the base and hypotenuse of a right triangle of which the altitude is 9.3 inches and the hypotenuse is 31.7 inches.

7. The altitude of a certain right triangle is 2.13 inches and the angle between the base and hypotenuse is 12°30′. Find the hypotenuse.

8. Which scales are used to find the sine of 20°?

9. Find the sine of arc cosine 0.75.

10. What is the vertical projection of a length of 1250 feet at an angle of 7°30′ with the horizontal?

Verify each of the following equations:

11. $\text{Sin } 40^\circ = 2 \sin 20^\circ \cos 20^\circ$

12. $\text{Sin } 55^\circ = 2 \sin 27^\circ 30' \cos 27^\circ 30'$

13. $\text{Sin } 15^\circ + \sin 35^\circ = 2 \sin 25^\circ \cos 10^\circ$

14. $\text{Sin } 60^\circ - \sin 20^\circ = 2 \cos 40^\circ \sin 20^\circ$

15. $\text{Sin } 45^\circ = \sin 10^\circ \cos 35^\circ + \cos 10^\circ \sin 35^\circ$

16. $\text{Sin}^2\ 67^\circ 30' + \cos^2 67^\circ 30' = 1$

17. $\text{Cos } 50^\circ = \cos 65^\circ \cos 15^\circ + \sin 65^\circ \sin 15^\circ$

18. $2 \sin 42^\circ 30' \cos 22^\circ 30' = \sin 65^\circ + \sin 20^\circ$

19. $\text{Cos } 48^\circ 20' + \cos 20^\circ = 2 \cos 34^\circ 10' \cos 14^\circ 10'$

20. $\cos 35° = 1 - 2 \sin^2 17°30'$

Find the sine of each of the following angles:

21. $20'30''$	**23.** $0°24'$	**26.** $30'$	**29.** $5'22''$
22. $1°46'$	**24.** $83°10'$	**27.** $86°$	**30.** $42°20'$
	25. $79°30'$	**28.** $89°$	

Find the arc sine of each of the following numbers:

31. 0.00342	**33.** 0.487	**35.** 0.633
32. 0.971	**34.** 0.185	

Find the cosine of each of the following angles:

36. $17°10'$	**38.** $64°45'$	**40.** $54°40'$
37. $85°22'$	**39.** $40°40'$	

CHAPTER X

THE TANGENT OF AN ANGLE

The tangent of an angle appears in many problems you want to solve with the slide rule. You will find it very convenient to be able to determine the tangent of an angle from the slide rule, and not be forced to consult a set of tables. The operation can be performed rapidly and the value you obtain is precise enough for most practical calculations.

THE TANGENT OF AN ANGLE. The problem is to find the tangent of an angle. The angle is expressed in degrees and minutes, and the tangent is a number. The tangent scale is on the back of the slide and is designated by the letter *T* at the right end. Start the operation by pulling the slide to the right and turning the slide rule over so you can see the tangent scale. Then adjust the slide so the angle on the tangent scale is under the mark* on the celluloid insert. Next turn the slide rule over again so you can see the front of it. Read the tangent of the angle on the C scale over the right index of the D scale. For the purpose of this calculation, the C scale may be considered to extend from *0.1* at the left index to *1* at the right index. Here is a word of caution. Remember there are 60′ in 1°. Keep this in mind when you read the tangent scale.

> **Rule 6. (a) The Tangent.** *Set the angle on the tangent scale under the mark on the celluloid insert. Read the tangent of the angle on the C scale above the right index of the D scale.*

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*If there is no mark on the celluloid insert, use the right edge of the celluloid to mark the angle. On those slide rules which do not have a celluloid insert, you will find a mark on the wood. Use this.

ILLUSTRATIVE EXAMPLES

1. Find the tangent of 30°.

1) Pull the slide to the right and turn the slide rule over.
2) Adjust the slide so that *30°* on the tangent scale is under the mark on the celluloid insert. Fig. 40 shows this setting of the slide.
3) Turn the slide rule over again and read the tangent of *30°* on the C scale over the right index of the D scale. The tangent of

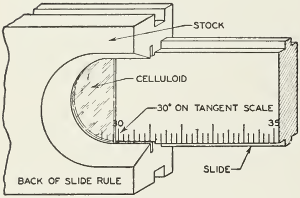

Fig. 40

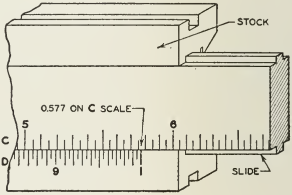

Fig. 41

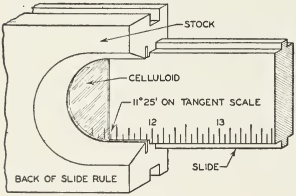

Fig. 42

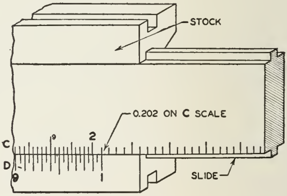

Fig. 43

30° is *0.577*. Fig. 41 shows part of the front of the rule for this reading.

2. Find the tangent of 11°25′.

1) Pull the slide to the right and turn the rule over.
2) Bring *11°25′* on the tangent scale under the mark on the celluloid as shown in Fig. 42. Notice that each space between *11°* and *12°* on the tangent scale represents *5′* of angle.

3) Turn the slide rule over again and read the tangent of *11°25′* as *0.202* on the C scale over the right index of the D scale. This setting is shown in Fig. 43.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

Find the tangent of each of the following angles:

1. 33°45′	6. 5°55′
2. 26°30′	**7.** 20°15′
3. 17°25′	8. 12°48′
4. 42°30′	9. 37°30′
5. 8°37′	**10.** 18°45′

11. The altitude of a right triangle is equal to the base times the tangent of the angle between the base and the hypotenuse. Find the altitude of a right triangle which has a base of 137.5 feet and in which the angle between the base and hypotenuse is 22°45′.

12. Find the base of a right triangle of which the altitude is 4.85 inches and the angle between the base and hypotenuse is 41°20′.

Verify each of the following equations:

13. $\text{Tan } 55° = \dfrac{\tan 30° + \tan 25°}{1 - \tan 30° \tan 25°}$

14. $\text{Tan } 45° = \dfrac{2 \tan 22°30'}{1 - \tan^2 22°30'}$

15. $\text{Tan } 15° = \dfrac{\tan 35° - \tan 20°}{1 + \tan 35° \tan 20°}$

Basis of the Process. The C scale is a logarithmic scale in that the distance from the left index to the location of a particular number represents the logarithm of the number. The tangent scale is laid out to correspond with the C scale so that the distance from its left end to a particular angle represents the logarithm of the tangent of the angle. Thus when the slide is set to find the tangent of an angle, this relation is true,

logarithm of number on C scale = logarithm of tangent of angle

If the logarithms of two quantities are equal, the two quantities are equal and so,

number on C scale = tangent of angle

THE TANGENT OF A VERY SMALL ANGLE. The tangent of an angle less than *5°43′* cannot be obtained in the usual way because the angle cannot be set on the tangent scale. However, a special method is available for such angles.

When the Angle Is between 1° and 5°43′. If the angle is between 1° and 5°43′, the first step is to convert it to a number of degrees plus a decimal fraction. Thus *2°30′* is *2.5°*, since *30′* is one-half of 1°. The general procedure here is to divide the number of minutes by 60 and attach it to the number of degrees as a decimal fraction. Thus, for the angle, *1°50′*, the number *50* is divided by *60.* The result is *0.833* and this is attached to *1°* giving *1.833°* for the angle. When the angle is obtained in this form, it is divided by *57.3.* The result is a close approximation of the tangent of the angle, so close that it suffices for practically all calculations.

ILLUSTRATIVE EXAMPLES

3. Find the tangent of 3°34′.

1) Convert *34′* to a decimal fraction of 1° by dividing by 60. The result is *0.567°*.
2) The angle *3°34′* is *3.567°*.
3) Divide *3.567* by *57.3.* The result is *0.0623* and this is the tangent of *3°34′*.

4. Find the tangent of 1°17′.

1) Convert *17′* to a decimal fraction of 1° by dividing by 60. The result is *0.283°*.
2) The angle *1°17′* is *1.283°*.
3) Divide *1.283* by *57.3.* The result is *0.0224.* The tangent of *1°17′* is *0.0224.*

A guide in locating the decimal point is that the tangent of *1°* is *0.0175* and the tangent of *5°43′* is *0.1.* Thus if the angle is between *1°* and *5°43′,* the tangent of the angle is between *0.0175* and *0.1.*

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

Find the tangent of each of the following angles:

1. 5°30′	**3.** 3°22′	6. 2°25′	9. 3°56′
2. 1°51′	**4.** 4°	**7.** 1°15′	**10.** 2°11′
	5. 4°41′	8. 5°05′	

11. An automobile travels on a 2°45′ grade. The horizontal projection of the distance traveled is 2830 feet. What is the change in altitude?

12. Find the base of a right triangle of which the altitude is 0.375 inches and the angle between the base and hypotenuse is 1°17′.

When the Angle Is Less Than 1°. An angle less than 1° would ordinarily be expressed in minutes and seconds, as *32′20″*. The first step in the process is to convert the number of seconds to a decimal fraction of 1′ by dividing by 60. Then attach this decimal fraction to the number of minutes. For example, *20* divided by *60* is *0.333*, so *32′20″* is *32.333′*. When you have the angle expressed in this form, divide it by *3440*. The result may be used as the tangent of the angle. A guide to the location of the decimal point is the fact that the tangent of *1°* is *0.01746*. The tangent of an angle less than *1°* is less than *0.01746*.

ILLUSTRATIVE EXAMPLES

5. Find the tangent of 25′32″.

1) Divide *32* by *60*. The result is *0.533*.
2) The angle *25′32″* is *25.533′*.
3) Divide *25.533* by *3440*. The result is *0.00742*. Hence the tangent of *25′32″* is *0.00742*.

6. Find the tangent of 0′53″.

1) Divide *53* by *60*. The result is *0.883*.
2) The angle *0′53″* is *0.883′*.
3) Divide *0.883* by *3440*. The result is *0.000257*, so the tangent of *0′53″* is *0.000257*.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

Find the tangent of each of the following angles:

1. 15′20″	**3.** 8′12″	6. 0′18″	**9.** 17′23″
2. 55′45″	**4.** 23′15″	7. 12′25.5″	**10.** 59′30″
	5. 45′	8. 38′30″	

Basis of the Process. The tangent of a very small angle is approximately equal to the angle in radians. One radian is 57.3°, so if the angle in degrees is divided by 57.3, the result is the angle in radians. This may be used as the tangent.

If the angle is expressed in minutes it must be divided by 60 in order to obtain it in degrees. Then it can be divided by 57.3 to obtain the tangent. The entire process is equivalent to dividing by 60×57.3, or 3440.

THE TANGENT OF AN ANGLE BETWEEN 45° AND 84°17′. The tangent of an angle between 45° and 90° is greater than one. It is found by subtracting the angle from 90° and setting the result on the tangent scale in the usual way. The tangent of the original angle is read on the D scale under the left index of the C scale. For this purpose the D scale is thought of as ranging from *1* at the left index to *10* at the right index.

ILLUSTRATIVE EXAMPLES

7. Find the tangent of 65°.

1) Subtract *65°* from *90°*. The result is *25°*.
2) Pull the slide to the right and turn the slide rule over.
3) Adjust the slide so that *25°* on the tangent scale is under the mark on the celluloid insert.
4) Turn the slide rule over again and read the tangent of *65°* on the D scale under the left index of the C scale. It is *2.15*.

8. Find the tangent of 82°30′.

1) Subtract *82°30′* from *90°*. The result is *7°30′*.
2) Pull the slide to the right and turn the slide rule over.
3) Adjust the slide so that *7°30′* on the tangent scale is under the mark on the celluloid.

4) Turn the slide rule over and read the tangent of *82°30′* as *7.60* on the D scale under the left index of the C scale.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

Find the tangent of each of the following angles:

1. 46°30′	3. 83°46′	6. 77°45′	9. 80°37′
2. 72°15′	4. 50°	7. 63°28′	**10.** 84°17′
	5. 59°25′	8. 45°	

Basis of the Process. In order to find the tangent of an angle θ, you set $(90° - \theta)$ on the tangent scale. You have, then, the tangent of $(90° - \theta)$, which we will call x, on the C scale over the right index of the D scale. The number, call it y, on the D scale under the left index of the C scale is the result of dividing one by x. That is, y is the reciprocal of x. As an equation,

$$y = \frac{1}{x}$$

If you substitute, $y = \tan\theta$, and $x = \tan(90° - \theta)$, the equation becomes,

$$\tan\theta = \frac{1}{\tan(90° - \theta)}$$

This last formula can be verified in any book on trigonometry.

THE TANGENT OF AN ANGLE BETWEEN 84°17′ AND 90°. When the angle is greater than 84°17′, different measures must be used. First subtract the angle from 90°. This will usually give a result in degrees and minutes. The number of minutes should be converted to a decimal fraction of a degree by dividing by 60. Then this decimal fraction is attached to the number of degrees. When 57.3 is divided by this number of degrees, the result is the tangent of the original angle.

ILLUSTRATIVE EXAMPLES

9. Find the tangent of 86°30′.

1) Subtract *86°30′* from *90°*. The result is *3°30′*.

2) Convert *30′* to a decimal fraction of a degree by dividing by *60*. This gives *0.5*, so *3°30′* is equal to *3.5°*.
3) Divide *57.3* by *3.5*. The result is *16.36*, so *16.36* is the tangent of *86°30′*.

10. Find the tangent of 88°48′.

1) Subtract *88°48′* from *90°*. This gives *1°12′*.
2) Convert *12′* to a decimal fraction of a degree. Do this by dividing *12* by *60*. The result is *0.2*, so *1°12′* is *1.2°*.
3) Divide *57.3* by *1.2*. This gives *47.7* which is the tangent of *88°48′*.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers in the back of the book.

Find the tangent of each of the following angles:

1. 85°	**3.** 89°15′	**6.** 87°22′	**9.** 86°55′
2. 85°57′	**4.** 84°45′	**7.** 88°34′	**10.** 87°12′
	5. 86°15′	**8.** 89°10′	

Basis of the Process. The basis of this process is the equation,

$$\tan \theta = \frac{1}{\tan (90° - \theta)}$$

The problem is to find $\tan \theta$. If the angle θ is greater than 84°17′, $(90° - \theta)$ is less than 5°43′, so $\tan (90° - \theta)$ is less than 0.1. Hence,

$$\tan (90° - \theta) = \frac{90° - \theta}{57.3}$$

This was explained under Basis of Process for finding the tangent of an angle less than 1°. Inverting each side of the equation,

$$\frac{1}{\tan (90° - \theta)} = \frac{57.3}{90° - \theta}$$

Therefore,

$$\tan \theta = \frac{57.3}{90° - \theta}$$

THE TANGENT OF AN ANGLE GREATER THAN 90°. The tangent of an angle greater than 90° can

always be expressed in terms of the tangent of an angle less than 90°. Then the tangent of the angle less than 90° can be found directly from the slide rule.

Angles between 90° and 180°. For any angle θ which is between 90° and 180°, the following equation is true:

$$\tan \theta = -\tan (180° - \theta)$$

After subtracting θ from 180°, the tangent of $(180° - \theta)$ is found by the methods discussed earlier.

11. Find the tangent of 157°30′.

1) Tan *157°30′* = − tan (*180° − 157°30′*) = − tan *22°30′*.
2) The tangent of *22°30′* is *0.413*. Hence, the tangent of *157°30′* is − *0.413*.

12. Find the tangent of 103°.

1) Tan *103°* = − tan (*180° − 103°*) = − tan *77°*.
2) The tangent of *77°* is *4.33,* so the tangent of *103°* is − *4.33*.

Angles between 180° and 270°. When the angle θ is between 180° and 270°, the tangent is found by using the equation,

$$\tan \theta = \tan (\theta - 180°)$$

13. Find the tangent of 235°30′.

1) Tan *235°30′* = tan (*235°30′ − 180°*) = tan *55°30′*.
2) The tangent of *55°30′* is *1.456*. Therefore, the tangent of *235°30′* is *1.456*.

14. Find the tangent of 187°45′.

1) Tan *187°45′* = tan (*187°45′ − 180°*) = tan *7°45′*.
2) The tangent of *7°45′* is *0.136,* so the tangent of *187°45′* is *0.136*.

Angles between 270° and 360°. If the angle θ is between 270° and 360°, the proper equation to use is,

$$\tan \theta = -\tan (360° - \theta)$$

15. Find the tangent of 298°20′.

1) Tan *298°20′* = − tan (*360° − 298°20′*) = − tan *61°40′*.
2) The tangent of *61°40′* is *1.857,* so the tangent of *298°20′* is − *1.857*.

16. Find the tangent of 323°30′.

1) Tan *323°30′* = − tan (*360° − 323°30′*) = − tan *36°30′*.
2) The tangent of *36°30′* is *0.739*. Hence, the tangent of *323°30′* is − *0.739*.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

Find the tangent of each of the following angles:

1. 227°20′	**3.** 171°35′	**6.** 145°	**9.** 207°30′
2. 132°15′	**4.** 346°10′	**7.** 225°	**10.** 315°
	5. 301°30′	**8.** 99°50′	

THE ARC TANGENT OF A NUMBER BETWEEN 0.1 AND 1. The arc tangent of a number is the angle which has the number for its tangent. The arc tangent is expressed in degrees and minutes. When the number is between 0.1 and 1 the process of finding the arc tangent starts with setting the slide so that the number on the C scale is over the right index of the D scale. Then the slide rule is turned over and the angle is read on the tangent scale under the mark on the celluloid insert.

> **Rule 6. (b) The Arc Tangent.** *Set the number on the C scale above the right index of the D scale. Read the angle which is the arc tangent of the number on the tangent scale under the mark on the celluloid insert.*

ILLUSTRATIVE EXAMPLES

17. Find arc tangent 0.645.

1) Adjust the slide so that *0.645* on the C scale is over the right index of the D scale.
2) Turn the slide rule over and read arc tangent *0.645* on the tangent scale under the mark on the celluloid. Arc tangent *0.645* is *32°52′*.

18. Find arc tangent 0.203.

1) Set the slide so that *0.203* on the C scale is over the right index of the D scale.

2) Turn the slide rule over and read the arc tangent of *0.203* as *11°30′* on the tangent scale under the mark on the celluloid.

It is well to keep in mind that if the tangent of the angle is between 0.1 and 1, the angle is between 5°43′ and 45°.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

Find the arc tangent of each of the following numbers:

1. 0.118
2. 0.953
3. 0.866
4. 0.333
5. 0.707
6. 0.693
7. 0.101
8. 0.467
9. 0.198
10. 0.300

THE ARC TANGENT OF A NUMBER BETWEEN 1 AND 10. The arc tangent of a number between 1 and 10 is found by setting the left index of the C scale over the number on the D scale. The angle which is on the tangent scale under the mark on the celluloid is then subtracted from 90°. The result is the arc tangent of the number. Follow these examples on your own slide rule.

ILLUSTRATIVE EXAMPLES

19. Find arc tangent 2.5.

1) Set the left index of the C scale to *2.5* on the D scale.
2) Turn the slide rule over and read the angle on the tangent scale which is under the mark on the celluloid insert. This angle is *21°50′*.
3) Subtract *21°50′* from *90°*. The result is *68°10′* and this is arc tangent *2.5*.

20. Find arc tangent 1.234.

1) Adjust the slide so that the left index of the C scale is over *1.234* on the D scale.
2) Turn the slide rule over and read the angle on the tangent scale under the mark as *39°*.
3) Subtract *39°* from *90°*. The result is *51°*. Hence, *51°* is arc tangent *1.234*.

21. Find arc tangent 8.48.

1) Set the left index of the C scale to *8.48* on the D scale.

2) Turn the slide rule over and read the angle on the tangent scale under the mark on the celluloid. This angle is *6°44′*.
3) Subtract *6°44′* from *90°*. The result, *83°16′*, is arc tangent *8.48.*

If the tangent of an angle is between 1 and 10, the angle must be between 45° and 84°17′.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

Find the arc tangent of each of the following numbers:

1. 1.012
2. 1.500
3. 3.46
4. 5.77
5. 4.02
6. 6.67
7. 1.853
8. 2.13
9. 3.07
10. 1.350

THE ARC TANGENT OF A NUMBER LESS THAN 0.1. If the tangent of an angle is less than 0.1, the angle can be found by multiplying the number by 57.3. The result gives the angle in degrees, and usually this will be an integral number of degrees plus a decimal fraction of a degree. This decimal fraction can be converted to minutes by multiplying by 60, since there are 60′ in 1°.

ILLUSTRATIVE EXAMPLES

22. Find arc tangent 0.0782.

1) Multiply *0.0782* by *57.3*. The result is *4.48,* so the arc tangent of *0.0782* is *4.48°*.
2) Convert *0.48°* to minutes by multiplying by *60*. The answer is *28.8*, and *0.48°* is *28.8′*, or, to the nearest minute, *29′*.
3) Arc tangent *0.0782* is *4°29′*.

When the angle is greater than 1°, it is usually sufficient to obtain the result to the nearest minute.

23. Find arc tangent 0.0376.

1) Multiply *0.0376* by *57.3*. This gives *2.16*. Hence, the angle is *2.16°*.
2) Convert *0.16°* to minutes by multiplying by *60*. The result is *9.6,* so *0.16°* is equal to *9.6′*. To the nearest minute, this is *10′*.
3) Arc tangent *0.0376* is *2°10′*.

If the angle is less than 1°, you may want to express it in minutes and seconds. When the number is multiplied by 3440, the result is the angle in minutes. Any decimal fraction attached to it can be converted to seconds by multiplying by 60, since there are 60″ in 1′. If the tangent is less than 0.01746, the angle must be less than 1°.

24. Find arc tangent 0.0137.

1) Multiply *0.0137* by *3440*. The result is *47.2,* so arc tangent *0.0137* is *47.2′*.
2) Convert *0.2′* to seconds. Do this by multiplying *0.2* by *60*. The result is *12,* so *0.2′* is equal to *12″*.
3) Arc tangent *0.0137* is *47′12″*.

25. Find arc tangent 0.000862.

1) Multiply *0.000862* by *3440*. This gives *2.97*. Hence, arc tangent *0.000862* is *2.97′*.
2) Convert *0.97′* to seconds by multiplying *0.97* by *60*. The result is *58.2,* so *0.97′* is equal to *58.2″*. To the nearest second this is *58″*.
3) Arc tangent of *0.000862* is *2′58″*.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

Find the arc tangent of each of the following numbers:

1. 0.088	**3.** 0.097	**6.** 0.00105	**9.** 0.0073
2. 0.00281	**4.** 0.048	**7.** 0.00193	**10.** 0.025
	5. 0.01103	**8.** 0.066	

THE ARC TANGENT OF A NUMBER GREATER THAN 10. If the tangent of an angle is greater than 10, the angle must be between 84°17′ and 90°. The first step in finding the angle is to divide 57.3 by the number. Regard this as an angle in degrees and subtract it from 90°. The result is the angle which has the original number for its tangent. The angle obtained by dividing 57.3 by the number will usually contain a decimal fraction. It is best to convert this into minutes by multiplying by 60.

ILLUSTRATIVE EXAMPLES

26. Find arc tangent 23.1.

1) Divide *57.3* by *23.1*. The result is *2.48,* that is *2.48°*.
2) Convert *0.48°* to minutes by multiplying by *60*. The result is *28.8,* so *0.48°* is equal to *28.8′*. To the nearest minute, this is *29′*. Hence *2.48°* is *2°29′*.
3) Subtract *2°29′* from *90°*. This gives *87°31′* which is arc tangent *23.1*.

27. Find arc tangent 37.5.

1) Divide *57.3* by *37.5*. The result is *1.529,* which is *1.529°*.
2) Convert *0.529°* to minutes. Do this by multiplying by *60*. The result is *31.8,* or *31.8′*. To the nearest minute, this is *32′*, so *1.529°* is equal to *1°32′*.
3) Subtract *1°32′* from *90°*. This gives *88°28′* which is arc tangent *37.5*.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

Find the arc tangent of each of the following numbers:

1. 10.75	**3.** 17.32	**6.** 11.08	**9.** 57.3
2. 42.3	**4.** 20.1	**7.** 31.4	**10.** 75
	5. 29.7	**8.** 21.9	

Find the tangent of each of the following angles:

11. 17°30′	**13.** 5°52′	**16.** 28°30′	**19.** 31°
12. 65°40′	**14.** 43°25′	**17.** 2°45′	**20.** 86°10′
	15. 77°10′	**18.** 81°17′	

Find the arc tangent of each of the following numbers:

21. 0.362	**23.** 0.085	**26.** 21.5	**29.** 0.854
22. 3.62	**24.** 0.747	**27.** 0.167	**30.** 0.0552
	25. 1.525	**28.** 2.72	

Verify each of the following equations:

31. $\text{Tan } 40° = \dfrac{\tan 25° + \tan 15°}{1 - \tan 25° \tan 15°}$

32. $\text{Tan } 75° = \dfrac{\tan 55° + \tan 20°}{1 - \tan 55° \tan 20°}$

33. $\text{Tan } 22°30' = \dfrac{\tan 45° - \tan 22°30'}{1 + \tan 45° \tan 22°30'}$

34. $\text{Tan } 20° = \dfrac{\tan 18° + \tan 2°}{1 - \tan 18° \tan 2°}$

35. $\text{Tan } 5°55' = \dfrac{\tan 37°55' - \tan 32°}{1 + \tan 37°55' \tan 32°}$

36. $\text{Tan } 5°55' = \dfrac{\tan 83°55' - \tan 78°}{1 + \tan 83°55' \tan 78°}$

OTHER TYPES OF SLIDE RULES. The tangent scale is always in the same location on the Mannheim slide rule. When the slide rule is turned over so that you can see the back, the tangent scale appears on the lower edge of the back of the slide. However, there is one rather common type of slide rule in which the slide

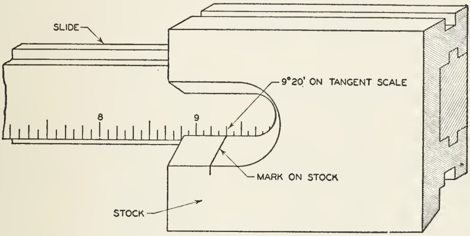

Fig. 44

must be extended to the left when the tangent scale is being used. There is a mark on the left end of the back of the stock which serves to locate the angle on the tangent scale. This is seen in Fig. 44, where the angle *9°20′* is set at the mark. To find the tangent of the angle, the slide rule is turned over again and the tangent is read on the C scale over the left index of the D scale. The tangent of *9°20′* is *0.1645*. To find the tangent of an angle larger than 45°, it is necessary to set, on the tangent scale, the difference between 90° and the angle, that is, 90° minus the angle. The tangent is then read on the D scale under the right index of the C scale. (If the angle is greater than 45°, the tangent of the angle must be greater than 1.) For example, in order to find the tangent of *55°*, it would be necessary to set *90° — 55°*, or *35°*, on the tangent scale. The tangent of *55°* is read as *1.427* on the D scale under the right index of the C scale.

REVIEW PROBLEMS

Answers to Review Problems are not given in the back of the book. Readers who are working alone may check their answers by looking up the tangent or arc tangent in a set of tables.

Here is another opportunity to learn by practice.

1. The base of a certain right triangle is 52.3 feet in length and the angle between the base and hypotenuse is $12°10'$. Find the altitude.

2. A right triangle has an altitude of 7.93 inches and base of 6.17 inches. What is the angle between the base and the hypotenuse?

3. Find the tangent of arc sine 0.62.

4. Which scales are used in finding the tangent of an angle?

5. The sun's rays, at an angle of $33°$ with the horizontal, cast the shadow of a building to a distance of 285 feet from the base of the building. How high is the building?

Find the tangent of each of the following angles:

6. $19°50'$	**8.** $40°30'$	**11.** $4°25'$	**14.** $73°10'$
7. $83°47'$	**9.** $57°45'$	**12.** $1°17'$	**15.** $27°15'$
	10. $6°32'$	**13.** $88°30'$	

Find the arc tangent for each of the following numbers:

16. 8.55	**18.** 0.1008	**21.** 0.075	**24.** 0.00345
17. 0.433	**19.** 1.940	**22.** 0.017	**25.** 0.948
	20. 0.693	**23.** 2.37	

Verify each of the following equations:

26. $$\text{Tan } 25° = \frac{\sin 50°}{1 + \cos 50°}$$

27. $$\text{Tan } 30° = \frac{\tan 20° + \tan 10°}{1 - \tan 20° \tan 10°}$$

28. $$\text{Tan } 34° = \frac{2 \tan 17°}{1 - \tan^2 17°}$$

29. $$\text{Tan } 16° = \frac{\tan 74° - \tan 58°}{1 + \tan 74° \tan 58°}$$

30. $$1 + \tan^2 75° = \frac{1}{\cos^2 75°}$$

CHAPTER XI

THE LOG SCALE

Many engineering formulae, for instance the formula for belt friction, contain the logarithm of a number. Also many calculations, such as raising a number to a fractional power, require the use of logarithms. For these reasons, it is desirable to be able to find the logarithm of a number from the slide rule, and also to be able to reverse this process; that is, to find the number which has a certain logarithm.

In speaking of a logarithm it is necessary to state the base of the logarithm; that is, the logarithm of a number to the base 10, or the base a, or the base b. The logarithm of a number to a certain base is the power to which the base must be raised to equal the number. Thus the logarithm of 1000 to the base 10 is three, since 10 must be raised to the power three to equal 1000. Any number can be used as a base for logarithms, but only two are used widely. In the common logarithm of a number, the base 10 is used. This is very convenient for calculation, as will be seen later. For the natural logarithm of a number, the base is a number called e. The value of e is 2.7183——. The greatest advantage of the base e is in work involving higher mathematics, calculus and beyond. You may not be interested in calculus, but you probably are interested in formulae which other people have derived by the use of calculus. Many of these formulae contain natural logarithms so you may have occasion to use them.

Both kinds of logarithms, common and natural, will be discussed in this chapter.

LOGARITHMS TO THE BASE 10. The logarithm of a number to the base 10 is the power to which 10 must be raised to equal the number. As an example, the logarithm of 154 to the base 10 is 2.1875. This means that *10* must be raised to the power

2.1875 to equal *154*. The *mantissa* of the logarithm is the part to the right of the decimal point; in this example, *1875*. The convenience of the base 10 is due to the fact that the mantissa of the logarithm of a number to the base 10 depends only upon the sequence of digits in the number and not at all upon the location of the decimal point in the number. Thus for the numbers *1.54, 154, 15400,* etc., the mantissa of the logarithm to the base 10 is always *1875*. The mantissa is the part of the logarithm that is found with the slide rule.

The part of the logarithm to the left of the decimal point is called the *characteristic* of the logarithm. When the base 10 is used, the characteristic of the logarithm is always one less than the digit count* for the number. Thus the characteristic of the logarithm of *154* is two, since the digit count for *154* is three. The characteristic can always be determined by inspection of the number and this is the first step in finding the logarithm to the base 10 with the slide rule.

To find the mantissa for a number, the number on the C scale is brought over the right index of the D scale. Then the slide rule is turned over so that the log scale, which is on the back of the slide, can be seen. The log scale is designated by the letter *L* at its right end. The mantissa of the logarithm to the base 10 is read on the log scale under the mark† on the celluloid insert. With both the characteristic and the mantissa known, the logarithm is completely determined. Try the process by following these examples on your own slide rule.

Rule 7. (a) The Log. *Set the number on the C scale above the right index of the D scale. Read the mantissa of the logarithm of the number on the log scale under the mark on the celluloid insert.*

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*Digit counts are explained on p. 50.

†Some slide rules do not have a mark on the celluloid insert. In such a case, use the right edge of the celluloid as a mark. If there is no celluloid insert at all, you will find a mark on the wood, and this mark will do.

ILLUSTRATIVE EXAMPLES

1. Find the logarithm of 2730 to the base 10.

1) The digit count for *2730* is four. The characteristic is one less than four, or three.
2) Set *2730* on the C scale over the right index of the D scale. Fig. 45 shows this setting of the slide.
3) Turn the slide rule over and read the mantissa of the logarithm of *2730* on the log scale under the mark on the celluloid insert. The mantissa is read as *436*. Fig. 46 shows a portion of the log scale for this reading.

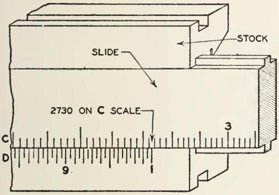


Fig. 45

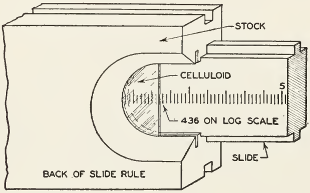


Fig. 46

4) The characteristic of the logarithm of *2730* is *3* and the mantissa is *436*. Put the characteristic to the left of the decimal point and the mantissa to the right. Then the logarithm of *2730* is *3.436*.

2. Find the logarithm of 7.36 to the base 10.

1) The digit count for *7.36* is one. The characteristic is one less than this, or zero.
2) Slide *7.36* on the C scale to a location directly above the right index of the D scale.
3) Turn the slide rule over and read the mantissa of the logarithm of *7.36* as *867* on the log scale under the mark on the celluloid insert.
4) The logarithm of *7.36* to the base 10 is *0.867*.

The log scale is to be thought of as starting with zero at the left end. Keep this in mind when the mantissa is very small.

3. Find the logarithm of 1,147,000 to the base 10.

1) There are seven digits to the left of the decimal point in *1,147,000,* so the digit count is seven. The characteristic is one less than seven, or six.
2) Set *1,147,000* on the C scale over the right index of the D scale.
3) Turn the slide rule over and read the mantissa for *1,147,000* as *060* on the log scale under the mark on the celluloid insert.
4) The logarithm of *1,147,000* to the base 10 is *6.060.*

It is necessary to use a special method in writing the characteristic for the logarithm of a number less than one. Such a logarithm is negative. The characteristic is in two parts, an integer in the usual position of the characteristic and — *10* following the mantissa. The integer is obtained by adding to 9 the digit count for the number. If the number is less than one, the digit count is either zero or negative. For example, the digit count for 0.000287 is minus three, since each zero immediately following the decimal point is counted a negative digit. To add a negative number is really to subtract*. Thus, to add — 3 to 9 would be,

$$9 + (-3) = 6$$

or,

$$9 - 3 = 6$$

When the entire logarithm is obtained, it is written as, *6.458 — 10.* This would be read as, (*6.458*) — *10,* the logarithm of *0.000287* to the base 10. A few examples will make the process clear.

ILLUSTRATIVE EXAMPLES

4. Find the logarithm of 0.00434 to the base 10.

1) The digit count for *0.00434* is minus two. The result of adding minus two to *9* is *7*. Thus the integer which is to occupy the usual position of the characteristic is *7*.
2) Bring *0.00434* on the C scale to a point directly above the right index of the D scale.
3) Turn the slide rule over and read the mantissa of the logarithm of *0.00434* as *638* on the log scale under the mark on the celluloid.

---

*Negative numbers are explained on p. 240.

4) Write down *7.638* and follow with (— *10*). Then the logarithm of *0.00434* to the base 10 is *7.638 — 10.*

5. Find the logarithm of 0.108 to the base 10.

1) The digit count is zero because there are zero digits to the left of the decimal point in *0.108.* Zero added to *9* leaves *9.*
2) Set *0.108* on the C scale above the right index of the D scale. Fig. 47 shows the left end of the C scale for this setting.
3) Turn the slide rule over and read the mantissa for *0.108* as *033* on the log scale under the mark on the celluloid. Fig. 48 shows part of the log scale for this reading.
4) Write *9.033* and attach — *10.* The logarithm of *0.108* to the base 10 is *9.033 — 10.*

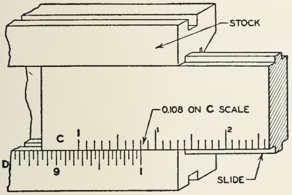


Fig. 47

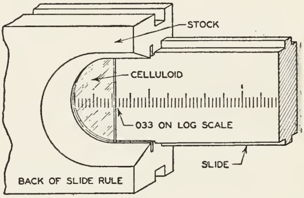


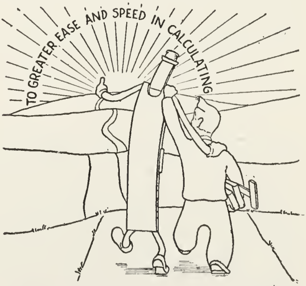
Fig. 48

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

Find the logarithm of each of the following to the base 10:

1. 10380
2. 52.3
3. 0.0708
4. 2.43
5. 0.607
6. 135.2
7. 1.973
8. 29,600,000
9. 0.0000065
10. 872



THE SLIDE RULE WILL HELP YOU DO IT

Basis of the Process. You will remember from the discussions of the previous chapters that the C scale is laid out so that the distance from its left index to the location of a particular number represents the mantissa of the logarithm of the number to the base 10. The log scale, as can be seen by inspecting it, is laid out so that the distance from its left end to the location of a particular number represents the magnitude of the number. Thus when the two scales are properly aligned, the following relation is true,

mantissa of logarithm of number to base 10, on
C scale = number on log scale

This equation shows the process to be valid.

THE ANTILOG FOR A GIVEN LOGARITHM TO THE BASE 10. In the use of many formulae containing logarithms, you obtain first the logarithm of the answer, then you must find the number corresponding to this logarithm. This number is called the *antilog*. It can be found easily with the slide rule for logarithms to the base 10. The process starts with the mantissa of the logarithm, which is the part of the logarithm to the right of the decimal point. (In the logarithm *3.172,* the mantissa is *172.*) Pull the slide to the right and turn the slide rule over. Next, adjust the slide so that the mantissa on the log scale is under the mark on the celluloid insert. Then turn the slide rule over again and read the antilog on the C scale over the right index of the D scale. Write down the digits of this number in their proper order. That is all you can get from the mantissa of the logarithm of the number to the base 10. You cannot tell from the mantissa where to locate the decimal point in the antilog, since the mantissa depends only on the digits of the number. For example, the mantissa is the same for the numbers *23.40, 234.0, 2340,* etc., since the digits are the same in these numbers.

The location of the decimal point in the antilog for a given logarithm is determined from the characteristic of the logarithm. The characteristic is the part of the logarithm to the left of the decimal point of the logarithm. (In the logarithm *3.172,* the characteristic is *3.*) When the base for the logarithm of the number is 10, the characteristic is always one less than the digit count for the number.

In the case we are considering now, you know the logarithm and you want to find the number for it. After finding the digits in the number from the slide rule, just place the decimal point in the number so that *the digit count is one more than the characteristic.* Note that this is just the reverse of the process used in finding the characteristic. In this case we know the characteristic, so we add one to it. If the characteristic of the logarithm is one, there must be two digits to the left of the decimal point of the antilog. If the characteristic of the logarithm is four, there must be five digits to the left of the decimal point of the antilog; and so forth. Read the following examples carefully and check each operation with your own slide rule.

Rule 7. (b) The Antilog. *Set the mantissa of the logarithm on the log scale under the mark on the celluloid insert. Read the antilog on the C scale above the right index of the D scale.*

ILLUSTRATIVE EXAMPLES

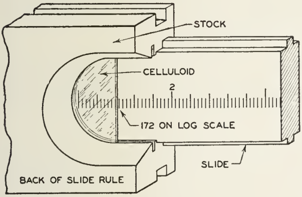


Fig. 49

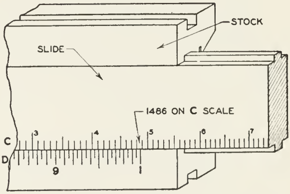


Fig. 50

6. Find the number which has 3.172 for its logarithm to the base 10. This is, find the antilog of 3.172.

1) Pull the slide to the right and turn the rule over.
2) Adjust the slide so that the mantissa, *172,* on the log scale is under the mark on the celluloid insert. Fig. 49 shows *172* under this mark.

3) Turn the slide rule over again and read the digits in the answer on the C scale over the right index of the D scale. The digits of the answer are *1486*. This is shown in Fig. 50.

4) The characteristic of *3.172* is *3*. The digit count for the answer must be one more than three, or four. Thus the answer is *1486.0*.

7. Find the number which has 1.589 for its logarithm to the base 10.

1) Pull the slide to the right and turn the rule over.
2) Bring *589* on the log scale under the mark on the celluloid.
3) Turn the slide rule over again. Read the digits in the answer as *388* on the C scale over the right index of the D scale.
4) Since the characteristic of the logarithm is *1*, the digit count for the number is two, and there must be two digits to the left of the decimal point in the answer. Hence, it is *38.8*.

8. Find the antilog for 0.027.

1) Pull the slide to the right and turn the rule over.
2) Set *027* on the log scale under the mark on the celluloid insert.
3) Turn the slide rule over again and read the digits in the answer as *1064* on the C scale over the right index of the D scale. If you do not read *1064*, it is because you have not set *027* correctly on the log scale. The mantissa *027* is between the numbers *0* and *1* on the log scale.
4) The characteristic of the logarithm is zero. The digit count for the answer must be one more than zero, or one. Therefore, the answer is 1.064.

If a number is less than one, its logarithm will have a special form, as $9.378 - 10$, or $7.917 - 10$. Consequently, when the logarithm is in such form, you know that the number (the antilog) is less than one. The mantissa is used in the ordinary way to find the digits in the number that corresponds to the logarithm. (The mantissa of $9.378 - 10$ is *378*. The mantissa of $7.917 - 10$ is *917*.) To locate the decimal point in the number which corresponds to the logarithm, subtract nine from the number in the usual position of the characteristic. (The number in the usual position of the characteristic is *9* for $9.378 - 10$. It is *7* for $7.917 - 10$.) The result

is zero or a negative number, and it is the digit count for the number corresponding to the logarithm. When the digit count is negative, you must insert zeros between the decimal point and the first digit of the number. Thus if the digits in the answer are *274* and the digit count is minus two, the answer is *0.00274*. Each zero to the right of the decimal point and before the *2* counts as a negative digit. Try this procedure in the following examples.

ILLUSTRATIVE EXAMPLES

9. Find the number which has 9.378 — 10 for its logarithm to the base 10.

1) Pull the slide to the right and turn the rule over.
2) Set *378* on the log scale under the mark on the celluloid insert.
3) Turn the slide rule over again. Read the digits in the answer as *239* on the C scale over the right index of the D scale.
4) The number in the usual position of the characteristic is *9*. Subtract nine from nine, obtaining zero. This is the digit count for the number and means that there are no digits to the left of the decimal point in the answer, so it is *0.239*.

10. Find the number which has 7.917 — 10 for its logarithm to the base 10.

1) Pull the slide to the right and turn the rule over.
2) Set the mantissa, *917,* on the log scale under the mark on the celluloid.
3) Turn the slide rule over again and read the digits in the answer as *826* on the C scale over the right index of the D scale.
4) The number in the usual position of the characteristic is *7*. Subtract nine from seven, which leaves minus two, which is the digit count for the answer. Then there are two zeros to the right of the decimal point in the answer and it is *0.00826*.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers in the back of the book.

These problems will help you to remember what you have just learned. Find the number corresponding to each of these logarithms to the base 10.

1. 2.828	**3.** 1.212	**6.** 8.195 — 10	**9.** 1.058
2. 0.545	**4.** 9.347 — 10	**7.** 4.308	**10.** 0.484
	5. 3.008	**8.** 9.950 — 10	

LOGARITHMS TO THE BASE *e*. The logarithm of a number to the base *e* is the power to which *e* must be raised to equal the number. The value of *e* is *2.7183*——. This base is very convenient for work in higher mathematics. Since many formulae used in engineering and shop work are derived by the use of higher mathematics, the logarithm of a number to the base *e* appears in many formulae. You cannot find the logarithm to the base *e* directly from the ordinary slide rule, but can find the logarithm to the base 10 and convert it to the base *e* by using this equation,

$$\text{logarithm of a number to the base } e = 2.3026 \times \text{logarithm of the number to the base } 10$$

First find the logarithm of the number to the base 10. Then multiply this logarithm by *2.3026** and you will have the logarithm of the number to the base *e*.

ILLUSTRATIVE EXAMPLES

11. Find the logarithm of 3.81 to the base *e*.

1) The logarithm of *3.81* to the base 10 is *0.581*.
2) Multiply *0.581* by *2.3026*. The result is *1.34*, which is the logarithm of *3.81* to the base *e*.

12. Find the logarithm of 145.4 to the base *e*.

1) The logarithm of *145.4* to the base 10 is *2.163*.
2) Multiply *2.163* by *2.3026*. The result is *4.98*, so the logarithm of *145.4* to the base *e* is *4.98*.

The logarithm of any number less than one is negative. This is true whether the base of the logarithm is 10 or *e*. The logarithm of *0.208* to the base 10 is,

$$9.318 - 10$$

In this case each part of the logarithm is to be multiplied by *2.3026* in order to find the logarithm to the base *e*. This would give,

$$21.47 - 23.026$$

*You will make this multiplication on the slide rule, using the C and D scales. The number *2.3026* will have to be set as *2.30* since you cannot represent the last two digits of the number on the slide rule.

When the indicated subtraction is carried out, the result is (-1.556). The logarithm of 0.208 to the base e is (-1.556).

13. Find the logarithm of 0.0871 to the base e.

1) The logarithm of *0.0871* to the base 10 is $8.940 - 10$.
2) Multiply $8.940 - 10$ by 2.3026. The result is $20.60 - 23.026$, or (-2.426). The logarithm of *0.0871* to the base e is (-2.426).

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers in the back of the book.

Find the logarithm of each of the following numbers to the base e.

1. 203	3. 37.3	6. 0.0569	9. 1.792
2. 5.42	4. 0.617	7. 137	**10.** 876
	5. 10	8. 20,000	

THE ANTILOG FOR A GIVEN LOGARITHM TO THE BASE *e*. In many calculations it is necessary to be able to find the number which has a given logarithm to the base e. This number is called the antilog. The easiest way to find it with the ordinary slide rule is to convert the logarithm of the number to the base e into the logarithm of the same number to the base 10. This conversion can be accomplished by dividing the logarithm to the base e by *2.3026*. Then when you know the logarithm of the number to the base 10, you can find the number by the method already given in this chapter.

ILLUSTRATIVE EXAMPLES

14. Find the number which has 3.05 for its logarithm to the base e.

1) Divide *3.05* by *2.3026*. The result is *1.324*.
2) *1.324* is the logarithm of the number to the base 10. The number which has this logarithm is 21.1.

15. Find the number which has 0.527 for its logarithm to the base e.

1) Divide *0.527* by *2.3026*. This gives *0.229*.

2) The logarithm of the answer to the base 10 is *0.229*. The answer is 1.694.

It is necessary to pay particular attention to cases when the logarithm of the number to the base *e* is negative. This will be the case for any number less than one. When such a logarithm to the base *e* is divided by *2.3026,* the result must also be a negative number. For example, if the logarithm of a number to the base *e* is (— *0.783*), the logarithm of the number to the base 10 is equal to (— *0.783*) divided by *2.3026,* which is (— *0.340*). However, this logarithm to the base 10 is not expressed in the form to which you are accustomed. You must change it to the usual form by adding ten and subtracting ten. First add ten, which in this case would give *9.660,* since

$$10 + (-0.340)$$

can be rewritten as,

$$10 - 0.340 = 9.660$$

Then subtract ten and leave the result in this form:

$$9.660 - 10$$

This is the standard form for the logarithm of a number less than one to the base 10. The method of finding the number corresponding to it has already been described.

Do not confuse this with the process explained on page 196. Here you are converting a logarithm from the form in which it is written as — *0.340* to the form in which it is written as *9.660 — 10.* On page 196 you were finding the logarithm of the number *0.000287.*

ILLUSTRATIVE EXAMPLES

16. Find the number which has (— 1.932) for its logarithm to the base *e*.

1) Divide (— *1.932*) by *2.3026.* The result is (— *0.838*).
2) Add ten to (— *0.838*). This is,

$$10 + (-0.838)$$

which can be written as,

$$10 - 0.838 = 9.162$$

3) Subtract ten from *9.162* and leave the result as,

$$9.162 - 10$$

4) The logarithm of the answer is *9.162 — 10* to the base 10. The answer is 0.1452.

17. Find the number which has (— 5.52) for its logarithm to the base e.

1) Divide (— *5.52*) by *2.3026*. The result is (— *2.40*).
2) Add ten to (— *2.40*). This is,

$$10 + (-2.40)$$

or,

$$10 - 2.40 = 7.60$$

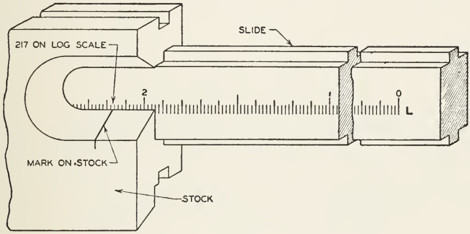


Fig. 51

3) Subtract ten from *7.60* and leave it as *7.60 — 10*.
4) The logarithm of the answer to the base 10 is *7.60 — 10*. The answer is 0.00398.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

Regard each of the following quantities as the logarithm of a number to the base e and find the number.

1. 4.18	**3.** 1.272	**6.** — 2.3026	**9.** — 0.667
2. 0.715	**4.** — 3.30	**7.** 2.950	**10.** 3.72
	5. 2.3026	**8.** 1.787	

OTHER TYPES OF SLIDE RULES. The log scale is usually in the center of the back of the slide on a Mannheim slide rule. However, on some slide rules the log scale is marked so that it must be read from right to left. This is shown in Fig. 51, where

217 on the log scale is set opposite the mark on the stock. Since *217* is the mantissa of a logarithm, the number corresponding is read on the D scale under the left index of the C scale. (The mantissa is the part of the logarithm to the right of the decimal point of the logarithm. Thus *217* is the mantissa of *2.217*. The characteristic is the part of the logarithm to the left of the decimal point of the logarithm. In the logarithm, *2.217,* the characteristic is *2*). The sequence of digits in this number on the D scale is *1647*. The characteristic of the logarithm determines the location of the decimal point in the number corresponding to the logarithm. The digit count for the number is one more than the characteristic of the logarithm of the number. The characteristic of the logarithm *2.217* is *2,* so the digit count for the number must be 3, there are three digits to the left of the decimal point in the number, and it is *164.7*.

REVIEW PROBLEMS

Answers to Review Problems are not given in the back of the book. Readers who are working alone may check their answers if desired by using tables of logarithms.

Find the logarithm of each of the following numbers to the base 10.

1. 15.6	**5.** 25,600	**9.** 0.00382	**13.** 0.283
2. 832	**6.** 96.7	**10.** 193.5	**14.** 327
3. 0.757	**7.** 58.5	**11.** 5280	**15.** 64.4
4. 6.28	**8.** 1.254	**12.** 4.43	

Find the number corresponding to each of the following logarithms to the base 10.

16. 3.462	**20.** 1.018	**24.** 1.633	**28.** 5.393
17. 1.789	**21.** 8.242 — 10	**25.** 1.995	**29.** 1.847
18. 0.317	**22.** 2.052	**26.** 0.447	**30.** 0.618
19. 2.549	**23.** 9.163 — 10	**27.** 0.500	

CHAPTER XII

THE RECIPROCAL SCALE

Many slide rules have a CI scale, or reciprocal scale. This scale is on the front of the slide and is designated by the letters *CI* at its left end. It differs from the C scale in that it reads from right to left instead of from left to right. Otherwise it is identical with the C scale.

For a given position of the runner on the slide rule, the number on the CI scale that is under the hairline is the reciprocal* of the number on the C scale that is under the hairline. (It is for this reason that the CI scale is often called the reciprocal scale.) This is illustrated by the following examples.

ILLUSTRATIVE EXAMPLES

1. Find the reciprocal of 4.

1) Set the hairline of the runner to *4* on the C scale. The slide can be in any position for this setting.
2) Read the reciprocal of *4* on the CI scale under the hairline. The reciprocal of *4* is 0.25.

2. Find the reciprocal of 17.

1) Set the hairline of the runner to *17* on the C scale.
2) Read the reciprocal of *17* on the CI scale under the hairline. The reciprocal of *17* is 0.0589.

3. Find the reciprocal of 0.65.

1) Set the hairline of the runner to *0.65* on the C scale.
2) Read the reciprocal of *0.65* as *1.54* on the CI scale under the hairline.

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*The reciprocal of a number x is $\frac{1}{x}$. Thus the reciprocal of 2 is $\frac{1}{2}$, the reciprocal of 5 is $\frac{1}{5}$, the reciprocal of 30 is $\frac{1}{30}$, etc.

4. Find the reciprocal of 0.866.

1) Set the hairline of the runner to *0.866* on the C scale.
2) Read the reciprocal of *0.866* as *1.154* on the CI scale under the hairline.

Advantages of using the CI scale are that it enables many problems to be done with fewer movements of the slide, and in many cases it makes possible a more precise calculation. These advantages will be discussed in greater detail in subsequent examples.

The CI scale can be used with the D scale for multiplication and division. In doing so, however, you must keep in mind when you set the hairline of the runner to a number on the CI scale, that the number with which you are actually calculating is its reciprocal.

MULTIPLICATION WITH THE CI SCALE. The problem is to multiply one number, the multiplicand, by a second, the multiplier, using the D scale and the CI scale.

STEPS IN THE PROCESS—MULTIPLICATION WITH CI SCALE

1) Set the hairline of the runner to the first number, or multiplicand, on the D scale.
2) Move the slide so that the second number, or multiplier, on the CI scale is under the hairline.
3) Read the answer on the D scale under whichever index of the C scale is between the ends of the D scale.
4) Locate the decimal point in the answer. Use the rules given in the following paragraph.

Be very careful in Step 2 that you read the CI scale from right to left. This may require care after your experience in using the other scales of the slide rule.

Locating the Decimal Point. Locate the decimal point by using one of the following rules. Only one can apply for a particular calculation.

1. *If the slide extends to the right of the stock during the calculation, the digit count* for the answer is exactly the sum of the*

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*Digit counts are explained on p. 50.

digit count for the multiplicand and the digit count for the multiplier.

2. *If the slide extends to the left of the stock during the calculation, the digit count for the answer is one less than the sum of the digit count for the multiplicand and the digit count for the multiplier.*

The following examples will demonstrate the entire process.

ILLUSTRATIVE EXAMPLES

5. Use the CI scale to multiply 18.8 by 6.15. Fig. 52 shows the proper setting of the slide and runner for this problem. It should be done in the following steps.

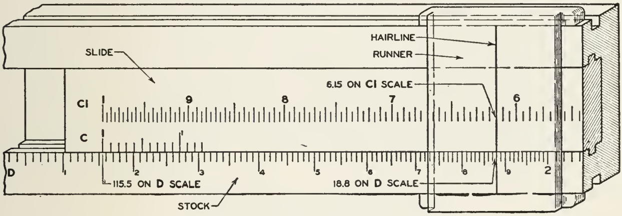


Fig. 52

1) Set the hairline of the runner to *18.8* on the D scale.
2) Move the slide so that *6.15* on the CI scale is under the hairline.
3) Read the numerals in the answer as *1155* on the D scale under the left index of the C scale.
4) The digit count for *18.8* is two, and the digit count is one for *6.15*. The sum of two and one is three. The slide extends to the right of the stock during the calculation so three is the digit count for the answer, which is *115.5*.

6. Use the CI scale to multiply 718 by 133.

1) Set the hairline of the runner to *718* on the D scale.
2) Move the slide so that *133* on the CI scale is under the hairline.
3) Read the numerals in the answer as *954* on the D scale under the right index of the C scale.

4) The digit count is three for each of the numbers *718* and *133*. Three plus three is six. Since the slide extends to the left of the stock, you are to subtract one from six, leaving five. This is the digit count for the answer, so it is *95400*.

In a decimal fraction such as *0.00438,* remember that each zero between the decimal point and the first digit of the number counts as a negative digit. Thus, the digit count for *0.00438* is minus two.

7. Use the CI scale to multiply 0.00438 by 0.0503.

1) Set the hairline of the runner to *0.00438* on the D scale.
2) Bring *0.0503* on the CI scale under the hairline.
3) Read the numerals of the answer as *22* on the D scale under the left index of the C scale.
4) The digit count for *0.00438* is minus two, and minus one for *0.0503*. The sum of minus two and minus one is minus three. Since the slide extends to the right of the stock, there is nothing to be subtracted from this sum and the digit count for the answer is minus three. This means that there must be three zeros between the decimal point and the first digit of the answer, so the answer is *0.00022*.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book. Perform each multiplication by using the CI scale.

1. $1.53 \times 6.92 = ?$ **2.** $45.5 \times 17 = ?$ **3.** $0.707 \times 58 = ?$

4. $0.023 \times 6560 = ?$

5. $8.4 \times 1230 = ?$

6. $0.693 \times 0.521 = ?$

7. $10.20 \times 73.5 = ?$

8. $9.98 \times 11.3 = ?$

9. $37.5 \times 0.866 = ?$

10. $0.402 \times 0.197 = ?$

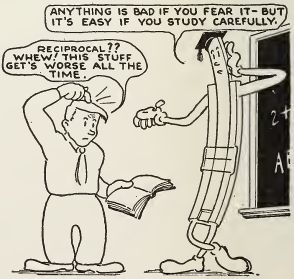
11. $56.4 \times 4900 = ?$

12. $804 \times 119 = ?$

13. $386 \times 212 = ?$

14. $184.8 \times 0.775 = ?$

15. $62.4 \times 313 = ?$



ALL YOU HAVE TO DO IS STUDY

Basis of the Process. Whenever we set out to explain why a certain slide rule manipulation gives the correct answer, we must resort to logarithms. The scales of the ordinary slide rule are based on logarithms to the base 10. The logarithm of a number to the base 10 is the power to which 10 must be raised to equal the number. Thus the logarithm of *16* to the base 10 is *1.2041* since 10 must be raised to the power *1.2041* to equal *16*. The part of the logarithm to the left of the decimal point is called the *characteristic* and is one less than the digit count for the original number. (Since there are two digits to the left of the decimal point in *16,* the digit count is two and the characteristic of its logarithm is *1*.) The part of the logarithm to the right of the decimal point is called the *mantissa* and it depends only on the sequence of digits in the number. Thus, the mantissa is *2041* for the numbers *16, 160, 1600,* etc.

Multiplication of two numbers can be accomplished by adding the logarithms of the two numbers, and this sum of the logarithms of the two numbers is the logarithm of the product of the two numbers. As an example, suppose we use logarithms to multiply *16* by *12*. The characteristic of each logarithm is *1,* since the digit count for each number is two. The mantissa for each can be found from a table of logarithms. Then

$$\begin{aligned} \text{Logarithm of } 16 &= 1.2041 \\ \text{Logarithm of } 12 &= \underline{1.0792} \\ \text{Sum} \ldots\ldots\ldots\ldots &= 2.2833 \end{aligned}$$

The logarithm of the product of *16* and *12* is *2.2833*. The mantissa of the logarithm of the product is *2833* and a table of logarithms shows that the sequence of numbers in the product must be *192*. The characteristic is *2,* so there must be three digits to the left of the decimal point. Hence, the answer is *192*.

The mantissa is the only part of the logarithm that can be represented on the slide rule. The distance from the left index of the D scale to the location of a number represents the mantissa of the logarithm of the number. However, on the CI scale, the distance from the right index to the location of a number represents the mantissa of the logarithm of the number. If we multiply *16* by *12,* using the CI scale, it will be evident that we have actually added

the mantissae for the two numbers. The problem would be done in the following steps:

1) Set the hairline of the runner to *16* on the D scale.
2) Bring *12* on the CI scale under the hairline.
3) Read the answer, *192,* on the D scale under the right index of the C scale.

Fig. 53 shows the proper setting of the slide and runner for this problem. It also shows that the operation actually adds the mantissae of *16* and *12* to give the mantissa of *192*. The slide rule gives the sequence of digits in the answer by using mantissae. The decimal point can be located by using characteristics. When the slide

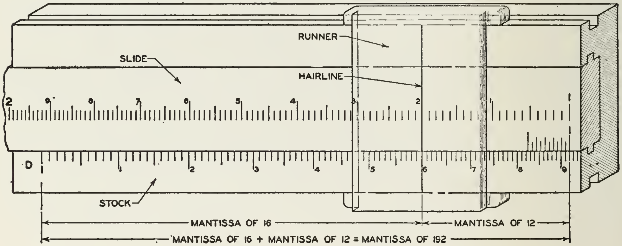


Fig. 53

extends to the left of the stock during such a problem, the characteristics add without any carry-over from the mantissae. Thus the characteristic of the product of two numbers is equal to the characteristic of the multiplicand plus the characteristic of the multiplier. As an equation, this is

characteristic of product = characteristic of multiplicand + characteristic of multiplier

Since the characteristic for a number is one less than the digit count for the number, this equation can be changed to

(digit count for product − 1) = (digit count for multiplicand − 1) + (digit count for multiplier − 1)

On each side a − 1 can be canceled, leaving

digit count for product = (digit count for multiplicand + digit count for multiplier) − 1

Hence the rule that when the slide extends to the left of the stock during a multiplication with the CI scale, the digit count for the product is equal to one less than the sum of the digit count for the multiplicand and the digit count for the multiplier.

When the slide extends to the right of the stock during such a problem, there is a carry-over of 1 from the mantissae to the characteristics when adding the logarithms. Thus the characteristic of the product is one more than the sum of the characteristics of the multiplicand and multiplier. As an equation, this is

characteristic of product = (characteristic of multiplicand + characteristic of multiplier) + 1

When this equation is expressed in terms of digit counts, it becomes

(digit count for product − 1) = (digit count for multiplicand − 1) + (digit count for multiplier − 1) + 1

All the 1's can be canceled, leaving

digit count for product = digit count for multiplicand + digit count for multiplier

Thus the rule, that when the slide extends to the right of the stock during a multiplication with the CI scale, the digit count for the product is equal to the sum of the digit count for the multiplicand and the digit count for the multiplier.

A shorter justification of the process of multiplication with the CI scale can be given by comparing it with the ordinary process of division (see chapter on Division). The comparison will show that in multiplying one number by a second with the CI scale, you are really dividing the first number by the reciprocal of the second. This gives the same result as multiplying the first number by the second.

DIVISION WITH THE CI SCALE. One number can be divided by a second by using the D scale and the CI scale. The first number is called the dividend and the second the divisor. The problem is worked in four steps.

STEPS IN THE PROCESS—DIVISION WITH CI SCALE

1) Place one index of the C scale to the dividend on the D scale.
2) Set the hairline of the runner to the divisor on the CI scale.
3) Read the answer on the D scale under the hairline.

4) Locate the decimal point.

Remember in carrying out Step 2 that the CI scale must be read from right to left.

Location of the Decimal Point. Here are the rules for locating the decimal point in the answer. They give the digit count for the answer.

1. *If the slide extends to the right of the stock, the digit count for the answer is equal to the digit count for the dividend minus the digit count for the divisor.*

2. *If the slide extends to the left of the stock, the digit count for the answer is one more than the digit count for the dividend minus the digit count for the divisor.*

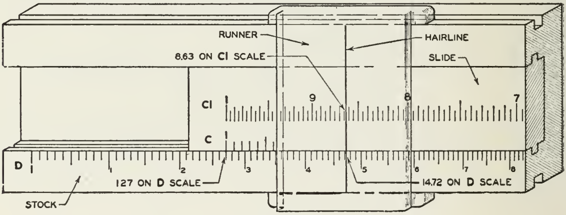


Fig. 54

ILLUSTRATIVE EXAMPLES

8. Use the CI scale to divide 127 by 8.63. Fig. 54 shows the proper setting of the slide and runner for this problem.

1) Set the left index of the C scale over *127* on the D scale.
2) Set the hairline of the runner to *8.63* on the CI scale.
3) Read the digits in the answer as *1472* on the D scale under the hairline.
4) The digit count is three for the dividend, *127,* and one for the divisor, *8.63*. Three minus one is two. Since the slide extends to the right of the stock, there is nothing to add to two, and it is the digit count for the answer. Hence the answer is *14.72*.

9. Use the CI scale to divide 0.062 by 5.27.

1) Set the right index of the C scale to *0.062* on the D scale.

2) Set the hairline of the runner to *5.27* on the CI scale.
3) Read the numerals in the answer as *1175,* on the D scale under the hairline.
4) In making a digit count for such a number as *0.062,* each zero immediately following the decimal point is counted as a negative digit. Thus the digit count is minus one for *0.062,* the dividend. The digit count is one for the divisor, *5.27*. One subtracted from minus one leaves minus two.* The slide projects to the left of the stock when solving the problem so one must be added to minus two, leaving minus one as the number of digit count for the answer. This means that there must be one zero immediately following the decimal point, so the answer is *0.01175.*

You must be careful in starting such a division to use the proper index of the C scale to locate the dividend on the D scale. If you use the wrong index, the divisor on the CI scale will not be between the ends of the D scale, and so you cannot complete the operation. When this happens, you must start over, using the other index of the C scale. The following example shows how this works.

10. Use the CI scale to divide 33.7 by 1.17.

1) Set the left index of the C scale to *33.7,* the dividend, on the D scale.
2) Try to set the hairline of the runner to *1.17,* the divisor, on the CI scale. You cannot because *1.17* on the CI scale is beyond the end of the D scale.
3) Therefore, set the right index of the C scale to *33.7* on the D scale.
4) Set the hairline of the runner to *1.17* on the CI scale.
5) Read the answer on the D scale under the hairline. It is *28.8.*

It is always possible to divide one number by another using the CI scale. If starting with the right index of the C scale will not let you complete the problem, starting with the left index will. One or the other will always work.

Sometimes the ordinary process of division, using the C and D

*Negative numbers are explained on p. 240.

scales (see chapter on Division for this method), results in an awkward setting of the slide rule. This is demonstrated in Example 11.

11. Divide 96 by 10.65 using the C and D scales. Fig. 55 shows the setting of the slide and runner for this problem. The steps in the operation are:

1) Set the hairline of the runner to *96* on the D scale.
2) Bring *10.65* on the C scale under the hairline.

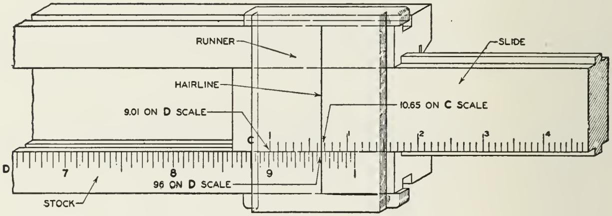


Fig. 55

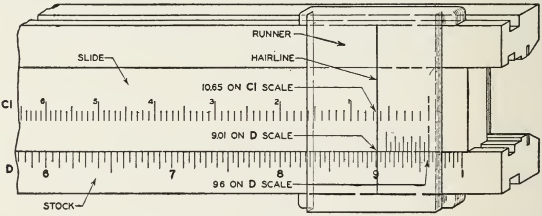


Fig. 56

3) Read the answer on the D scale under the left index of the C scale. Notice that only a small part of the slide is held within the stock. In this position the slide is likely to wobble and may slip while you are trying to read the answer.

The situation encountered in Example 11 can be avoided by performing the division with the CI scale. Example 12 shows how this is done.

12. Use the CI scale to divide *96* by *10.65*. Fig. 56 shows

the setting of the slide and runner. Do the problem in the following steps:

1) Set the right index of the C scale to *96* on the D scale.
2) Set the hairline of the runner to *10.65* on the CI scale.
3) Read the answer as *9.01* on the D scale under the hairline. Notice that nearly all of the slide is within the stock, held securely so that it cannot wobble and is not likely to slip while you are reading the answer.

Using the CI scale relieves you of making calculations with an awkward position of the slide. This advantage is demonstrated by Example 12. This may seem a little thing, but mastery of enough little things results in real skill in using the slide rule.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book. Make each division by using the CI scale.

1. $5280 \div 22.7 = ?$
2. $201 \div 17.32 = ?$
3. $0.693 \div 4.91 = ?$
4. $83400 \div 397 = ?$
5. $8.07 \div 128 = ?$
6. $13 \div 93 = ?$
7. $2.77 \div 0.148 = ?$
8. $0.042 \div 3.84 = ?$
9. $0.707 \div 0.051 = ?$
10. $922 \div 305 = ?$
11. $19.32 \div 16 = ?$
12. $17{,}000 \div 29{,}600{,}000 = ?$
13. $0.345 \div 78.5 = ?$
14. $60.9 \div 23.7 = ?$
15. $112 \div 51.5 = ?$

Basis of the Process. Here come the logarithms again. A quotient is equal to a dividend divided by a divisor. The logarithm of the quotient is equal to the logarithm of the dividend minus the logarithm of the divisor. Suppose we divide *150* by *12,* using logarithms to the base 10. The mantissa of the logarithm of each number can be obtained from a table of logarithms. (The *mantissa* is the part of the logarithm to the right of the decimal point of the logarithm.) The characteristic of the logarithm of a number can be found by using the rule that it is always one less than the digit count for the number. (The *characteristic* is the part of the logarithm to the left of the decimal point of the logarithm. The charac-

teristic of *150* is *2* and that of *12* is *1*.) After finding the mantissae of the logarithms from a set of tables, we can write

$$
\begin{array}{lrl}
\text{Logarithm of} & 150 = & 2.1761 \\
\text{Logarithm of} & 12 = & \underline{1.0792} \\
\text{Difference} & \ldots\ldots = & 1.0969
\end{array}
$$

The logarithm of the quotient is *1.0969*. The mantissa is *0969* and the tables show that the digits of the quotient are *125*. Since the characteristic of the quotient is *1*, the digit count must be two. Hence the quotient is *12.5*.

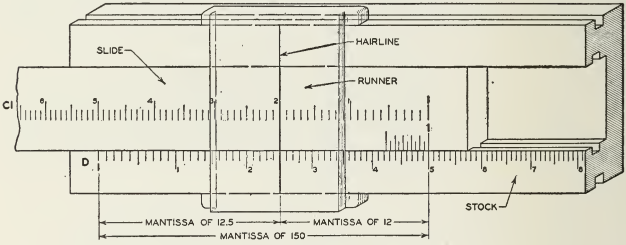


Fig. 57

On the slide rule, only the mantissa of the logarithm is represented. The distance from the left index of the D scale to the location of a number on the D scale represents the mantissa of the logarithm of the number. But on the CI scale, the distance from the right index to the location of a number represents the mantissa of the logarithm of the number. If *150* is divided by *12* on the slide rule, the operation actually subtracts the mantissa of *12* from the mantissa of *150* to give the mantissa of the quotient, *12.5*. The problem is done in the following steps, using the CI scale:

1) Set the right index of the C scale to *150* on the D scale.
2) Set the hairline of the runner to *12* on the CI scale.
3) Read the answer, *12.5*, on the D scale under the hairline.

Fig. 57 shows the setting of the slide and runner for this problem and demonstrates that the operation actually does subtract one mantissa from another.

Since only the mantissa of the logarithm of a number can be represented on the slide rule, and the mantissa represents only the sequence of digits of the number, only the sequence of digits in the answer can be read from the slide rule. However, the decimal point in the answer can be located by considering characteristics. If the slide projects to the left of the stock during a division in which the CI scale is used, the mantissae of the logarithms subtract without any carry-over to the characteristics. Thus the characteristics can be subtracted separately, and the characteristic of the quotient is equal to the characteristic of the dividend minus the characteristic of the divisor. As an equation, this is,

$$\text{characteristic of quotient} = \text{characteristic of dividend} - \text{characteristic of divisor}$$

This can be written in a more convenient form in terms of digit counts, remembering that the characteristic for a number is one less than the digit count for the number. Then

$$(\text{digit count for quotient} - 1) = (\text{digit count for dividend} - 1) - (\text{digit count for divisor} - 1)$$

or,

$$\text{digit count for quotient} - 1 = \text{digit count for dividend} - 1 - \text{digit count for divisor} + 1$$

After a (-1) is canceled from each side, there remains,

$$\text{digit count for quotient} = (\text{digit count for dividend} - \text{digit count for divisor}) + 1$$

Hence the rule that when the slide projects to the left of the stock during a division with the CI scale, the digit count for the quotient is one more than the digit count for the dividend minus the digit count for the divisor.

When the slide projects to the right beyond the stock during such a division, there is a carry-over of -1 from the mantissae to the characteristics in subtracting logarithms, and so the characteristic of the quotient is one less than the characteristic of the dividend minus the characteristic of the divisor. As an equation, this statement is,

$$\text{characteristic of quotient} = (\text{characteristic of dividend} - \text{characteristic of divisor}) - 1$$

When this equation is written in terms of digit counts, it becomes,

(digit count for quotient — 1) = (digit count for dividend — 1) — (digit count for divisor — 1) — 1

All of the — 1's will cancel, leaving,

digit count for quotient = digit count for dividend — digit count for divisor

And so, when the slide extends to the right of the stock during the process of division with the CI scale, the digit count for the quotient is equal to the digit count for the dividend minus the digit count for the divisor.

Another basis of this process can be given by comparing with the ordinary process of multiplication (see chapter on Multiplication). The comparison will show that when you use the CI scale to divide one number by a second, you are really multiplying the first number by the reciprocal of the second, which gives the same result as dividing the first number by the second.

MULTIPLICATION OF THREE NUMBERS. Three numbers can be multiplied conveniently by using the D, C, and CI scales in combination. The problem is to find the product of $a \times b \times c$. We will refer to the first number, *a,* as the multiplicand; to the second number, *b,* as the first multiplier; and to the third number, *c,* as the second multiplier. The procedure is to multiply *a* by *b* in the ordinary way, using the C and D scales, (see chapter on Multiplication), and then to multiply their product by *c* with the CI scale. The steps in the operation of the slide rule are:

1) Place one index of the C scale over the multiplicand on the D scale.
2) Multiply by the first multiplier by setting the hairline of the runner to the first multiplier on the C scale. The product of the multiplicand and the first multiplier is then located on the D scale under the hairline, but do not bother to write it down.
3) Multiply by the second multiplier by bringing the second multiplier on the CI scale under the hairline. This step multiplies the product of Step 2 by the second multiplier.
4) Read the answer on the D scale under whichever index of the C scale is between the ends of the D scale. Only one index of the C scale can be between the ends of the D scale at one time.

5) Locate the decimal point as described in the following paragraphs.

This process is efficient and will enable you to save time. It requires fewer movements of the slide and runner than the method described in the chapter on Multiplication.

Locating the Decimal Point. It is desirable to know a method for locating the decimal point in the answer. As you carry out the foregoing operations, apply the following rules:

1. *If the slide projects to the right of the stock in multiplying the multiplicand by the first multiplier with the C scale, jot down — 1. If the slide projects to the left, there is nothing to jot down.*

2. *If the slide projects to the right of the stock in multiplying by the second multiplier with the CI scale, there is nothing to jot down. If the slide projects to the left, jot down — 1.*

As you jot down the — 1's, carry them in a horizontal row, which will hereafter be called the *digit summation.* In a typical problem, you might have at this point, in the digit summation,

$$-1$$

As soon as you finish the manipulation of the slide rule, read the digits in the answer and write them down. Then return to the digit summation. Add to it the digit count for each of the original numbers of the problem. These original numbers are the multiplicand, the first multiplier, and the second multiplier. As an example, suppose they are *12.4, 275* and *5.13,* respectively. The digit count is two for the multiplicand, *12.4;* three for the first multiplier, *275;* and one for the second multiplier, *5.13.* The completion of the digit summation in this case would give,

$$-1+2+3+1=5$$

The final result of the digit summation is the digit count for the answer. A few examples will illustrate the entire process.

ILLUSTRATIVE EXAMPLES

13. Find the result of $580 \times 4.27 \times 32$.

1) Place the right index of the C scale over the multiplicand, *580,* on the D scale.
2) Multiply by the first multiplier, *4.27,* by setting the hairline of the runner to *4.27* on the C scale. Notice that the slide projects

to the left of the stock, so there is nothing to jot down for the digit summation.

3) Multiply by the second multiplier, *32,* by sliding *32* on the CI scale under the hairline. The slide projects to the left when this is done, so jot down — *1* in the digit summation.

4) Read the digits in the answer as *792* on the D scale under the right index of the C scale.

5) At this point the digit summation is,

$$-1$$

Add to it the digit count for each of the original numbers of the problem, *580, 4.27,* and *32.* The numbers to be added to the digit summation are, respectively, *3, 1,* and *2.* Thus the digit summation becomes,

$$-1+3+1+2=5$$

The final result, *5,* is the digit count for the answer. You must add two zeros to *792.* The result is *79,200.*

In the next example, there are numbers similar to 0.00224 in which there are several zeros after the decimal point and before the first digit of the number. Remember that each such zero is counted as a negative digit, so the digit count for *0.00224* is minus two.

14. Find the result of $0.0137 \times 6.67 \times 0.0048$.

1) Start by setting the left index of the C scale to the multiplicand, *0.0137,* on the D scale.

2) Multiply by the first multiplier, *6.67,* by setting the hairline of the runner to *6.67* on the C scale. The slide projects to the right beyond the stock when you have done this, so start the digit summation by jotting down — *1.*

3) Multiply by the second multiplier, *0.0048,* by sliding *0.0048* on the CI scale under the hairline. The slide projects to the right of the stock for this setting, so there is no contribution to the digit summation.

4) Read the digits of the answer as *439* on the D scale under the left index of the C scale.

5) At this point the digit summation is,

$$-1$$

Now add to the digit summation the digit count for each of the original numbers of the problem, *0.0137, 6.67* and *0.0048.* The numbers to add are respectively, $-$ *1, 1,* and $-$ *2.* This makes the digit summation

$$-1 + (-1) + 1 + (-2)$$

or,

$$-1 - 1 + 1 - 2 = -3$$

The final result, $-$ *3,* is the digit count for the answer, so you must insert three zeros after the decimal point and before the first digit of the answer, so it is *0.000439.*

Occasionally you will find it desirable to interchange the multiplicand and the first multiplier. For example, suppose the problem is to find the result of $91 \times 1.05 \times 32.2$. You would start by setting the left index of the C scale to *91* on the D scale, since *91* is the multiplicand. Then you would set the hairline of the runner to *1.05,* the first multiplier, on the C scale. But this is an awkward setting of the slide and runner, as you can see when you try it on your own slide rule. Most of the slide projects to the right of the stock so that it may wobble or slip in the stock. See how much better it is if you interchange *91* and *1.05.* Then you would start by setting the left index of the C scale over *1.05* on the D scale, using *1.05* as the multiplicand. Next you would set the hairline of the runner to the first multiplier, *91,* on the C scale. This setting is much better, since most of the slide remains within the stock and is held more securely. It is better to do the problem in the order $1.05 \times 91 \times 32.2$ than in the order $91 \times 1.05 \times 32.2$.

PRACTICE PROBLEMS

This procedure cannot be learned without practice. Do these problems carefully. Write out the digit summation for each one.

After you have worked all of the following problems, check your answers with the correct answers in the back of the book.

1. $22.5 \times 4.12 \times 47 = ?$

2. $1.93 \times 29 \times 0.707 = ?$

3. $0.866 \times 0.495 \times 558 = ?$

4. $6.28 \times 1024 \times 3 = ?$

5. $99 \times 88 \times 77 = ?$

6. $12.5 \times 0.071 \times 46 = ?$

7. $201 \times 0.0033 \times 3.97 = ?$

8. $5.21 \times 47 \times 0.618 = ?$

9. $3.14 \times 95 \times 0.206 = ?$
10. $0.0073 \times 4.62 \times 15.5 = ?$
11. $1.078 \times 301 \times 0.84 = ?$
12. $18.86 \times 0.595 \times 17 = ?$
13. $6.07 \times 291 \times 4.1 = ?$
14. $0.932 \times 7.8 \times 1.65 = ?$
15. $57.7 \times 0.0038 \times 2.5 = ?$

Basis of the Process. This process merely combines the ordinary method of multiplication (see chapter on Multiplication), with the method which uses the CI scale. The basis of each has been explained previously, so no further comments are necessary here.

DIVISION OF ONE NUMBER BY TWO OTHER NUMBERS. The CI scale can be used efficiently in combination with the C and D scales when dividing one number by two other numbers. Such an operation can be written as, $\frac{a}{b \times c}$. The number a is the dividend, b is the first divisor and c is the second divisor. The operation combines the ordinary method of division (see chapter on Division) with the method which uses the CI scale. It can be performed with the following manipulations of the slide rule:

1) Set the hairline of the runner to the dividend on the D scale.
2) Divide by the first divisor by bringing the first divisor on the C scale under the hairline.
3) Divide by the second divisor by setting the hairline of the runner to the second divisor on the CI scale.
4) Read the answer on the D scale under the hairline.

This process requires fewer settings of the slide and runner than the ordinary process which was explained in the chapter on Division.

Locating the Decimal Point. In many problems it would be difficult to locate the decimal point in the answer by inspection. It is desirable, therefore, to know a method for doing this. As you operate the slide and runner, follow these rules:

1. *When you divide the dividend by the first divisor, jot down + 1 if the slide projects to the right of the stock. If the slide projects to the left of the stock there is nothing to jot down.*

2. *When you divide by the second divisor, there is nothing to jot down if the slide projects to the right of the stock. If the slide projects to the left of the stock jot down + 1.*

Carry the + 1's in a horizontal line, as for instance,

$$+ 1 + 1$$

This line will hereafter be called the digit summation. As soon as you finish manipulating the slide rule, read the digits in the answer and write them down. Then return to the digit summation and complete it in order to know where to locate the decimal point in the answer. Add to the digit summation the digit count for the dividend. Subtract from it the digit count for each of the divisors. For example, if the problem is to find the result of

$$\frac{27.5}{1.24 \times 3.79}$$

you would add to the digit summation the digit count for the dividend, *27.5*. This is two, so the digit summation started above would become,

$$+ 1 + 1 + 2$$

Then you would subtract the digit count for each of the divisors, *1.24* and *3.79*. The digit count is one for each of these numbers, so you would subtract one twice, completing the digit summation as

$$+ 1 + 1 + 2 - 1 - 1 = 2$$

The final result of the digit summation, two in this case, is the digit count for the answer.

Follow the examples closely. This process is worth knowing.

ILLUSTRATIVE EXAMPLES

15. Find the result of $\dfrac{396}{19.5 \times 5.92}$

1) Set the hairline of the runner to the dividend, *396,* on the D scale.
2) Divide by the first divisor, *19.5,* by bringing *19.5* on the C scale under the hairline. The slide projects to the right beyond the stock during this setting. Hence, you must start the digit summation by jotting down + *1.*
3) Divide by the second divisor, *5.92,* by setting the hairline of the runner to *5.92* on the CI scale. The slide projects to the right beyond the stock so there is no contribution to the digit summation.

4) Read the digits of the answer as *344* on the D scale under the hairline.

5) At this point the digit summation is,

$$+1$$

Add to it the digit count for the dividend, *396*. This is three, so the digit summation becomes,

$$+1+3$$

Now subtract from the digit summation the digit count for each of the divisors, *19.5* and *5.92*. For these numbers, you subtract, respectively, two and one. This completes the digit summation as,

$$+1+3-2-1=1$$

The final result of the digit summation, one, is the digit count for the answer. Hence the answer is *3.44*.

Be careful when you are dealing with such a number as *0.00423,* which has a digit count of minus two. Remember that each zero immediately following the decimal point is counted as a negative digit.

16. Find the result of $\dfrac{2.12}{0.07 \times 0.0022}$

1) Start by setting the hairline of the runner to the dividend, *2.12,* on the D scale.

2) Divide by the first divisor, *0.07,* by bringing *0.07* on the C scale under the hairline. The slide projects to the left beyond the stock, so there is no contribution to the digit summation.

3) Divide by the second divisor, *0.0022,* by setting the hairline of the runner to *0.0022* on the CI scale. The slide projects to the left so start the digit summation by jotting down $+1$.

4) Read the digits of the answer as *1375* on the D scale under the hairline.

5) At this point the digit summation is,

$$+1$$

Add to it the digit count for the dividend, *2.12,* that is, add one. This gives,

$$+1+1$$

Now subtract from the digit summation, the digit count for each of the divisors, *0.07* and *0.0022*. The numbers to be subtracted are, respectively, minus one and minus two, since each zero after the decimal point and before the first digit of the number counts as a negative digit. This completes the digit summation as,

$$+1+1-(-1)-(-2)$$

or,

$$+1+1+1+2=5$$

Thus the digit count for the answer is five, so you must add a zero to *1375* and the answer is *13,750*.

Occasionally, when you are ready to divide by the second divisor, you will find that the second divisor on the CI scale is beyond the end of the D scale. When this happens, it is best to complete the problem by making the second division in the ordinary way (see chapter on Division). Move the hairline of the runner to whichever index of the C scale is within the ends of the D scale. Leave the runner in this position while sliding the second divisor on the C scale under the hairline. Read the answer on the D scale under whichever index of the C scale is between the ends of the D scale. Since each division in such a problem is done with the C scale, use the rule given in the chapter on Division for the calculations which lead to the location of the decimal point.

ILLUSTRATIVE EXAMPLE

17. Find the result of $\dfrac{54.4}{2.56 \times 1.3}$

1) Set the hairline of the runner to the dividend, *54.4,* on the D scale.
2) Divide by the first divisor, *2.56,* by sliding *2.56* on the C scale under the hairline. The slide projects to the right beyond the stock when you have done this, so start the digit summation with + *1*.
3) Try to divide by the second divisor, *1.3,* by setting the hairline of the runner to *1.3* on the CI scale. You cannot do this because *1.3* on the CI scale is beyond the right end of the D scale.

4) Therefore, move the hairline of the runner to the left index of the C scale.
5) Now divide by *1.3* by sliding *1.3* on the C scale under the hairline. The slide extends to the right beyond the stock so follow the first rule and add $+ 1$ to the digit summation.
6) Read the digits of the answer as *1637* on the D scale under the left index of the C scale. You know that you must read the answer under the left index of the C scale because the right index is beyond the end of the D scale.
7) At this point the digit summation is,

$$+1+1$$

Add to it the digit count for the dividend, *54.4*. This is two, which makes the digit summation

$$+1+1+2$$

Next subtract from the digit summation the digit count for each of the divisors, *2.56* and *1.3*. The digit count is one for each, so the digit summation is completed as

$$+1+1+2-1-1=2$$

This is the digit count for the answer. There are, then, two digits to the left of the decimal point in the answer, so it is *16.37*.

PRACTICE PROBLEMS

After you have worked all of the following problems, check your answers with the correct answers shown in the back of the book. Use the C scale for the first division and the CI scale for the second.

1. $\dfrac{74.8}{37.5 \times 5.30} = ?$

2. $\dfrac{1.81}{0.62 \times 1.19} = ?$

3. $\dfrac{14{,}500{,}000}{17{,}600 \times 732} = ?$

4. $\dfrac{0.000549}{0.0000065 \times 24} = ?$

5. $\dfrac{2.75}{4.3 \times 0.0178} = ?$

6. $\dfrac{61.4}{3.71 \times 47.5} = ?$

7. $\dfrac{29{,}600{,}000}{1.25 \times 9{,}800} = ?$

8. $\dfrac{120}{32.2 \times 11.1} = ?$

9. $\dfrac{768}{14.7 \times 6.14} = ?$

10. $\dfrac{39{,}300}{72 \times 51.5} = ?$

11. $\dfrac{938}{0.00812 \times 29} = ?$

12. $\dfrac{1.37}{82.7 \times 161} = ?$

13. $\dfrac{144}{17 \times 11} = ?$

14. $\dfrac{748}{9{,}500 \times 0.031} = ?$

15. $\dfrac{5.03}{0.044 \times 0.75} = ?$

Basis of the Process. This process combines the ordinary method of division (see chapter on Division) with the method which uses the CI scale. Each of these methods has been justified in previous pages, so no further explanation is given here.

PRACTICE PROBLEMS

Use the CI scale in making the calculations. *After* you have worked all of the following problems, check your answers with the correct answers shown in the back of the book.

1. $34.5 \times 27.3 = ?$
2. $0.947 \times 6.84 = ?$
3. $23.4 \times 1.78 \times 780 = ?$
4. $5280 \times 17 = ?$
5. $83.5 \times 0.682 = ?$
6. $3720 \div 6.28 = ?$
7. $0.866 \div 3.42 = ?$
8. $256 \div 13 = ?$
9. $14.4 \div 0.085 = ?$
10. $8.72 \div 2.7 = ?$
11. $\dfrac{5280}{45 \times 66} = ?$
12. $\dfrac{1750 \times 6.28}{60} = ?$
13. $\dfrac{43.5}{0.707 \times 0.386} = ?$
14. $\dfrac{28 \times 15 \times 12}{32.2 \times 17} = ?$
15. $\dfrac{65}{28.3 \times 1.64} = ?$
16. $37.7 \times 0.0243 = ?$
17. $15.6 \div 48.7 = ?$
18. $1.88 \times 63.5 \times 0.105 = ?$
19. $\dfrac{2.54}{6.28 \times 53} = ?$
20. $\dfrac{364{,}000}{1250 \times 4.17} = ?$

OTHER TYPES OF SLIDE RULES. On the slide rule on which this book is based the reciprocal scale is marked *CI*. On other slide rules, you may find it marked *R* (*R* standing for reciprocal). However, it will still be located on the slide and will be used in the manner described in this chapter.

REVIEW PROBLEMS

Answers to Review Problems are not given in the back of the book. Readers who are working alone may check their answers by working problems longhand or by using the C and D scales only.

Practice using the reciprocal scale on these problems. As you acquire experience, you will find that you work faster and with greater precision.

1. An orchard containing 112 pear trees produces 1350 bushels of pears. What is the average yield per tree?

2. A machine shop contracts to finish 520 gears at a price of 84 cents per gear. What is the price for the whole job?

3. Water weighs 8.33 pounds per gallon and there are 7.48 gallons of water in one cubic foot. How much will 2.7 cubic feet of water weigh?

4. A gross of a certain size of machine screw costs $3.12. How much does one dozen cost? (There are twelve dozen in one gross.)

5. A certain type of truck uses 0.22 gallons of gasoline per mile. How much gasoline will 17 such trucks use while each travels 484 miles?

6. In a construction gang, 75 men receive a total of $2710 for 35 hours, each, of work. What is the average hourly pay?

7. An airplane travels 410 miles in 2.3 hours. What is its average speed?

8. A contractor plans to pour 12 concrete foundations, each requiring 4.85 cubic yards of concrete. The concrete will cost $9.60 per cubic yard in place. What is the total cost?

9. A shop buys 120 iron castings, each weighing 5.35 pounds, for $42.60. What is the average cost per pound?

10. Storage space is required for 820 cubic feet of material. An area 9.5 feet wide is available, but the material cannot be stored to a greater depth than 7 feet. How long a space is needed?

11. $37.3 \times 0.0519 = ?$

12. $1.28 \div 60.9 = ?$

13. $9.7 \times 11.4 \times 0.805 = ?$

14. $5280 \times 19.5 \times 27 = ?$

15. $\dfrac{0.702}{2.17 \times 0.041} = ?$

16. $\dfrac{62.4}{3.14 \times 28} = ?$

17. $\dfrac{16{,}200}{258 \times 39.5} = ?$

18. $7.11 \times 493 \times 0.00145 = ?$

19. $0.866 \times 0.741 \times 28 = ?$

20. $\dfrac{913}{0.64 \times 0.437} = ?$

CHAPTER XIII

HOW TO CHECK A SLIDE RULE

No slide rule, or anything else for that matter, is perfect. Variations in materials and processes, and slight inaccuracies in workmanship, combine to produce an article that is short of perfection. The best of slide rules cannot be exactly right. However, the imperfections do not exist in the same degree in all slide rules. Some are better than others. It is with this in mind that this chapter is written. Before buying a slide rule it is advisable to check it carefully, so as to secure a rule that is quick and sure in operation and that will give results with precision.

Marking. Make sure that the scales of the slide rule stand out distinctly from one another, and that the marking is clear and easy to read. A great deal of time can be wasted and a great deal of eye strain suffered in trying to use a slide rule that cannot be read quickly and easily.

The Runner. Make sure that the runner slides easily so that it can be adjusted with the fingertips. There should be no rough spots noticeable as the runner is moved from one end of the rule to the other. Try moving it, first with the slide entirely within the stock, then with the slide extended to the left, and last with the slide extended to the right. It should move easily but should not be loose enough to wobble.

The Slide. The slide should move freely in the stock. There should be just enough resistance so that it will not fall out when the slide rule is held vertically. In many cases the slide will fit loosely in the stock when one end of the slide projects from the stock and tightly when the other end projects from the stock. This is not desirable. It should require about the same amount of force to push the slide, no matter what its position.

Pull the slide out of the stock and sight along both stock and slide to make sure that neither is warped. The stock is so much heavier than the slide that it will hold the slide straight when engaged, even though the slide is badly warped.

The C and D Scales. The C and D scales are used more than any others on the slide rule. For this reason they should be checked carefully. Place the left index of the C scale over the left index of the D scale and see if each mark on the C scale lines up with the corresponding mark on the D scale.

Try a few multiplications and divisions with the C and D scales and see if you get the correct answers. Use simple numbers so that you can check the answers mentally. For instance, see if *2* times *4* comes out exactly as *8,* if *7* divided by *2* is exactly *3.5,* etc.

The A and D Scales. Place the hairline of the runner over the left index of the D scale and see if it is over the left index of the A scale. Try this also with the right indices. Square a few simple numbers. See if the square of *8* comes out exactly as *64,* the square of *5* as *25,* the square of *3* as *9,* etc. If they do not, you don't want that slide rule.

The B and C Scales. Check the B and C scales together in the same way as the A and D scales.

The Sine Scale. Check the sine scale by finding the sine of each of several angles and comparing results with a table of trigonometric functions. For instance, make sure that you read the sine of $30°$ as *0.5,* the sine of $60°$ as *0.866,* etc. The last mark at the right end of the sine scale represents $90°$. See that the sine of $90°$ comes out as *1.*

The Tangent Scale. Try a few angles and compare with the tables. The tangent of $30°$ is *0.577,* and the tangent of $45°$ is *1,* etc. You cannot secure precise results with an inaccurate slide rule.

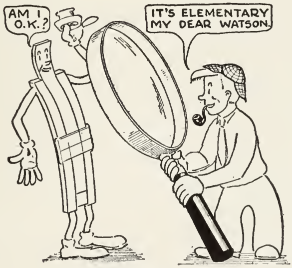
The Log Scale. All that can be obtained from the log scale is the mantissa of the logarithm of a number to the base 10, but try a few to make sure that you can obtain the mantissa accurately. The mantissa of *1* is *0,* that of *3* is *477,* that of *5* is *699.*

The K Scale. See that the left index of the K scale lines up with the left index of the D scale. You should be able to set the

hairline of the runner so that it will fall exactly on both indices at the same time. Try this also for the right indices. Try cubing a few simple numbers. Make sure that the cube of *2* is obtained exactly as *8,* the cube of *3* as *27,* the cube of *5* as *125,* etc.

The CI Scale. Set the hairline of the runner on the left index of the C scale and see that it falls exactly on the left index of the CI scale. Try this also for the right indices. Remember that when the hairline is set to a number on the C scale, the number on the CI scale that is under the hairline should be the reciprocal of the number on the C scale. See if you read the reciprocal of *2* as *0.5,* and that of *8* as *0.125.*

Conclusion. The satisfaction derived from the use of a precise and easily operated slide rule is well worth the small amount of time and effort required to select such a slide rule.



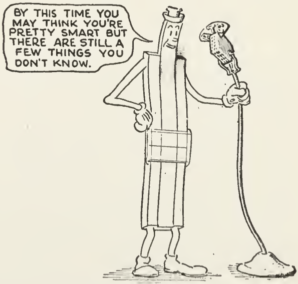
CHECK AND DOUBLE CHECK

CHAPTER XIV

FINAL SUGGESTIONS

If you have studied the earlier chapters carefully and practiced with a great many problems, you are now proficient in the use of the slide rule. You should be able to make any calculation in arithmetic with it. However, do not feel that you know all about the slide rule and that nothing more can be learned. Keep an open mind as you use it and try constantly to improve in speed and precision. Study the particular types of problems that you have to do and try to work out procedures for doing them more efficiently. The fundamental operations have been described in detail in this book and you should know them, however, it is for practical problems that you need the slide rule; and there is a great deal of difference between being able to do a problem, and being able to do it quickly and precisely.

Good Judgment. You can save time and avoid errors by exercising good judgment in the use of the slide rule. It is a good practice to make a rough mental check of each calculation to see if the answer is reasonable. One common mistake, especially for the beginner, is to set 201 as 210, or 101 as 110. Another, in the use of the C and D scales, is to set 2 as 12. This sort of error can usually be detected if you make a mental estimate of the answer and compare with the answer read from the slide rule.



AND IN CONCLUSION - - - -

Do not become such a slave to habit that you use the slide rule when you do not have to. For instance, do not use the slide rule to multiply 2 times 3, or 1.5 times 4, etc. Problems as easy as these can be done mentally.

Do not go through a procedure for locating the decimal point when the calculation is very simple. For instance, if you divide 31.7 by 6 and read the numerals of the answer as 528, you know that the decimal point must be located after the *5*. The answer must be *5.28*. It could not be 0.528 or 52.8. Save time where you can save it legitimately, and when it will not lead to errors.

Greater Precision. It is possible to read portions of the C and D scales more accurately than was done in earlier chapters. Fig. 58 shows the portion of the C scale between divisions *2* and *3*. So far you have been reading only three digits of an answer when it is located here, and this has involved estimating one-half a space when the third digit is odd. However, you can estimate much closer than this. Fig. 58 also shows an enlarged short length of the

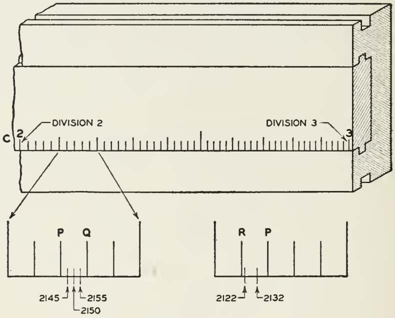


Fig. 58

C scale with correct locations for the numbers *2145, 2150* and *2155*. To locate these numbers requires estimating to one-fourth of a space. This can be done easily and enables you to obtain a more precise result. The mark designated by *P* represents *2140* and that designated by *Q* represents *2160*. The number *2150* is halfway between *2140* and *2160,* so it is located halfway from *P* to *Q*. The number *2145* is halfway between *2140* and *2150* so it is one-fourth of the way from *P* to *Q*. Also, since *2155* is halfway between *2150* and *2160,* it is located three-fourths of the way from *P* to *Q*.

Occasionally calculations will start with a number such as *2122*. The correct location for this number is shown at the lower right in Fig. 58. Note that this enlarged section shows the same part of the scale that is shown in the enlarged section on the left; namely, *2100* to *2200*. The mark designated by R represents *2120* and that designated by P represents *2140*. The number *2122* is one-tenth of the way from *2120* to *2140*, since the difference between 2120 and 2140 is 20 while the difference between 2120 and 2122 is 2. Thus, on the slide rule you would locate *2122* by estimating one-tenth of the space from *2120* to *2140*, here one-tenth of the space from R to P. This enlarged length also shows the correct location for the number *2132*. This number is twelve greater than 2120 so it must be located at twelve-twentieths, or six-tenths, of the space from R to P.

The type of marking between divisions *3* and *4* on the C scale is the same as that between divisions *2* and *3*. Hence, these remarks also apply to it. Note that portions of the A and B scales are similar, so you can estimate more closely there.

SUMMARY OF RULES FOR LOCATING THE DECIMAL POINT. The most commonly used rules for locating the decimal point in an answer are those for multiplication and division. Because of their importance, these rules are repeated here.

Multiplication with the C and D Scales.

1) *If the slide projects to the right of the stock during a multiplication with the C and D scales, the digit count for the product is one less than the sum of the digit counts for the multiplicand and multiplier.*

2) *If the slide projects to the left of the stock, the digit count for the product is equal to the sum of the digit counts for the multiplicand and multiplier.*

Multiplication with the CI and D Scales.

1) *If the slide projects to the right of the stock during a multiplication with the CI and D scales, the digit count for the product is equal to the sum of the digit counts for the multiplicand and multiplier.*

2) *If the slide projects to the left of the stock, the digit count for the product is one less than the sum of the digit counts for the multiplicand and multiplier.*

Division with the C and D Scales.

1) *If the slide projects to the right of the stock during a division with the C and D scales, the digit count for the quotient is one more than the digit count for the dividend minus the digit count for the divisor.*

2) *If the slide projects to the left of the stock, the digit count for the quotient is equal to the digit count for the dividend minus the digit count for the divisor.*

Division with the CI and D Scales.

1) *If the slide projects to the right of the stock during a division with the CI and D scales, the digit count for the quotient is equal to the digit count for the dividend minus the digit count for the divisor.*

2) *If the slide projects to the left of the stock, the digit count for the quotient is one more than the digit count for the dividend minus the digit count for the divisor.*

RULES FOR FUNDAMENTAL OPERATIONS ON THE SLIDE RULE

The rules for the fundamental operations of the slide rule are repeated below. They have been grouped together so that they will be convenient for reference and review. However, do not try to use this list of rules until you have studied the chapters in which they are explained and illustrated.

Rule 1. Multiplication. *Set one index of the C scale to the multiplicand on the D scale. Next, set the hairline of the runner to the multiplier on the C scale. Finally, read the answer on the D scale under the hairline.*

Rule 2. Division. *Set the hairline of the runner to the dividend on the D scale. Then slide the divisor on the C scale under the hairline. Finally, read the answer on the D scale under one index of the C scale.*

Rule 3. (a) The Square. *Set the hairline of the runner to the number on the D scale. Read the square of the number on the A scale under the hairline.*

(b) The Square Root. *Set the hairline of the runner to the number on the A scale. Read the square root of the number on the D scale under the hairline.*

Rule 4. (a) The Cube. *Set the hairline of the runner to the number on the D scale. Read the cube of the number on the K scale under the hairline.*

(b) The Cube Root. *Set the hairline of the runner to the number on the K scale. Read the cube root of the number on the D scale under the hairline.*

Rule 5. (a) The Sine. *Set the angle on the sine scale under the mark on the celluloid insert. Read the sine of the angle on the B scale under the right index of the A scale.*

(b) The Arc Sine. *Set the number on the B scale under the right index of the A scale. Read the angle which is the arc sine of the number on the sine scale under the mark on the celluloid insert.*

Rule 6. (a) The Tangent. *Set the angle on the tangent scale under the mark on the celluloid insert. Read the tangent of the angle on the C scale above the right index of the D scale.*

(b) The Arc Tangent. *Set the number on the C scale above the right index of the D scale. Read the angle which is the arc tangent of the number on the tangent scale under the mark on the celluloid insert.*

Rule 7. (a) The Log. *Set the number on the C scale above the right index of the D scale. Read the mantissa of the logarithm of the number on the log scale under the mark on the celluloid insert.*

(b) The Antilog. *Set the mantissa of the logarithm on the log scale under the mark on the celluloid insert. Read the antilog on the C scale above the right index of the D scale.*

NEGATIVE NUMBERS

A negative number is a number that is less than zero—sometimes called a *minus* number. Such numbers are designated by a *minus sign* in front of the number. This is never omitted. An example is minus three, written as *—3*. Minus three is three less than zero. Negative numbers are encountered in many types of mathematical calculations. They are nothing to worry about if you will study a few facts and rules about them.

Fig. A shows a vertical scale that illustrates the relationship between positive and negative numbers. The long mark at the center represents zero. All numbers above zero are plus, or positive, numbers. All numbers below zero are minus, or negative, numbers. Our limited space allows only a small part of the complete scale to be shown. The smallest possible number is negative and is at the very bottom of the complete scale. The largest possible number is positive and is at the very top of the complete scale. Look at Fig. A and note that any given number is less than any number above it, and is greater than any number below it. This is always true, regardless of the *numerical* or *absolute value** of the number. As examples of this, note by study of the scale that *5* is greater than *—7*; *0* is greater than *—3*; *—2* is greater than *—9*; *—7* is less than *—5*; *—2* is less than *2*; *—9* is less than *9*.

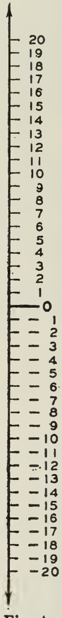


Fig. A

Addition and Subtraction—Using a Scale. To add to a number is always to increase its value. It follows then, that on the

*The *numerical* or *absolute* value of a number is its distance from zero. Example: although *—9* and *9* are not the same (see Fig. A), the absolute values of *—9* and *9* are the same (9 units) because each is the same distance from zero.

scale, Fig. A, *to add to a given number is to count up from the number.* Examples: (*1*) $5 + 6 = ?$ Starting at *5* we count up six spaces to stop on *11*; so $5 + 6 = 11$. (*2*) $-6 + 3 = ?$ Starting at *−6* we count up three spaces, to stop on *−3;* so $-6 + 3 = -3$.

To subtract from a number is always to decrease its value. Thus on the scale *to subtract from a given number is to count down from the number.* Examples: (*1*) $8 - 3 = ?$ Starting on *8,* we count down three spaces to stop on *5*; so $8 - 3 = 5$. (*2*) $3 - 4 = ?$ Starting on *3* we count down four spaces. In doing this we pass zero and stop on *−1;* so $3 - 4 = -1$. (*3*) $-6 - 4 = ?$ Starting on *−6* we count down four spaces to stop on *−10*; so $-6 -4 = -10$.

Addition and Subtraction by Rule.

RULE A. To find the sum of two or more numbers having the same sign, add the numbers arithmetically and prefix the common sign to the answer.

The following are examples of Rule A. Note that in each example both numbers have the same sign.

1. $6 + 3 = 9$	**4.** $-6 - 3 = -9$
2. $-4 - 7 = -11$	**5.** $4 + 7 = 11$
3. $2 + 5 = 7$	**6.** $-2 - 5 = -7$

RULE B. To find the sum of two numbers having unlike signs, subtract the number having the smaller numerical value from the number having the greater numerical value and prefix the sign of the greater number to the answer.

The following are examples of Rule B. Note that the two numbers in each example have unlike signs.

1. $-6 + 3 = -3$	**4.** $6 - 3 = 3$
2. $-5 + 3 = -2$	**5.** $5 - 3 = 2$
3. $-7 + 10 = 3$	**6.** $7 - 10 = -3$

Algebraic Sum. The answer obtained by following either of these two rules is called the *algebraic sum.* Note that adding two numbers in this way does not always give the sum of the numerical

values. For example, $-9+9=0$, not 18. Or, $-4+6=2$, not 10.

Addition of a Negative Number. When it is desired to add a negative number, first remove the parenthesis marks and replace the two original signs (to the left and right of the first parenthesis) by a negative sign, then proceed to find the algebraic sum of the two numbers. As an example, $6+(-3)=?$ Remove the parentheses, substituting a minus sign for the two signs (the plus sign to the left of the parenthesis and the minus sign within the parentheses), and you have $6-3=3$. Note the following examples:

1. $9+(-12)=-3$	5. $-9+(-7)=-16$
2. $16+(-3)=13$	6. $-15+(-4)=-19$
3. $14+(-8)=6$	7. $-3+(-6)=-9$
4. $7+(-12)=-5$	8. $-4+(-10)=-14$

Multiplication of Positive and Negative Numbers.

RULE C. The product of two numbers having like signs is positive. The product of two numbers having unlike signs is negative.

Division of Positive and Negative Numbers.

RULE D. The quotient obtained by dividing two numbers having like signs is a positive number. The quotient obtained by dividing two numbers having unlike signs is a negative number.

ANSWERS TO PRACTICE PROBLEMS

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1. See examples 1-5. **2.** 922. **3.** 478. **4.** 977. **5.** 1,021. **6.** 983. **7.** 453. **8.** 1,556 sq. ft. **9.** 3480. **10.** 255 mi. **11.** 264. **12.** 489 lb. **13.** $8. **14.** 1,256 lb. **15.** 783 lb.

Page 50

1. 999. **2.** 987. **3.** 1,027. **4.** 1,109. **5.** 1,118.

Page 53

1. 0.000347. **2.** 504. **3.** 0.00526. **4.** 14.87. **5.** 2,240,000. **6.** 0.0346. **7.** 6,720 ft. **8.** 1,925 cu. in. **9.** 11,620 lb. **10.** 63,300 in. **11.** 790.16. **12.** 2,625.

Page 55

1. 0.713. **2.** 0.00889. **3.** 94.2. **4.** 0.00892. **5.** 0.0000937. **6.** 0.000903. **7.** 5.36 gal. **8.** 8,070 lb. **9.** 740 sq. ft. **10.** 9,590,000 sq. ft.

Page 57

1. 0.818; 0.727; 0.687; 0.593; 0.477. **2.** 2,850; 6,490; 18,310; 27,300; 40,100; 68,200; 177,000; 230,000 cu. in. **3.** $1.00; $1.15; $1.30; $1.40; $1.62; $1.94. **4.** 32.3; 42.5; .44; 66; 73.3; 88; 105.6; 124.8; 139.3 ft. **5.** 110; 215; 328; 512; 678; 1,210 ft. **6.** 2.65; 3.55; 7.37; 12.1; 17.76; 22.6; 27.9 lb.

Page 60

1. 0.0366. **2.** 7,820. **3.** 1,900. **4.** 5.14. **5.** 6,880. **6.** 1,877 cu. ft. **7.** 2,990. **8.** 125.6 gal. **9.** 1.482. **10.** 2,080 cu. yd.

Page 64

1. 22.8. **2.** 43.3. **3.** 0.585. **4.** 0.858. **5.** 84,000. **6.** 0.189. **7.** 28.7. **8.** 72.5. **9.** 150,000. **10.** 9.54. **11.** 71. **12.** 1,015.

13. 35.4. **14.** 671,000. **15.** 20.3. **16.** 3,240 lb. **17.** 137.1 mi. **18.** 6,050. **19.** \$22.10. **20.** 1,069; 1,552; 490; 366; 62.8; 236. **21.** 11,250 sq. ft. **22.** 147,800 gal. **23.** 14.7 lb. per sq. in. **24.** 5,150. **25.** 9.3. **26.** 14.1. **27.** 21.3. **28.** 0.1036. **29.** 63.1. **30.** 213. **31.** 23.3. **32.** 63. **33.** 568. **34.** 70. **35.** 16.3. **36.** 17,440. **37.** 34,900. **38.** 55.7. **39.** 639. **40.** 91,200.

Page 69

1. 557. **2.** 173. **3.** 866. **4.** 136. **5.** 746. **6.** 72. **7.** 69¢. **8.** 361. **9.** 86¢. **10.** 798. **11.** \$32. **12.** 679. **13.** 768. **14.** 234 in. **15.** 12 ft.

Page 72

1. 1.203. **2.** 0.1952. **3.** 665. **4.** 0.1822. **5.** 15,750. **6.** 12,710. **7.** 153.3. **8.** 0.01322. **9.** 0.1141. **10.** 11.31.

Page 73

1. 0.00394. **2.** 9.22. **3.** 0.839. **4.** 0.0560. **5.** 0.000629. **6.** 17,680. **7.** 0.00858. **8.** 73.6. **9.** 0.0001101. **10.** 436.

Page 74

1. 0.636; 0.538; 0.467; 0.412; 0.368. **2.** 0.1428; 0.0769; 0.0667; 0.0476; 0.0323. **3.** 36.5; 27.2; 21.9; 15.48; 11.37.

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1. 0.1668; 0.257; 0.348; 0.469; 0.712; 0.864; 0.985. **2.** 0.1169; 0.1661; 0.239; 0.326; 0.482 cu. ft. **3.** 16; 29; 57; 69; 77; 97¢. **4.** 1.95; 33.8; 51.5; 80.0; 106.1; 12.42; 1.369. **5.** 13.65; 19.12; 26.7; 31.4; 43; 60; 75 m.p.h. **6.** 161; 371; 795; 106.8; 1,248; 13.12. **7.** 0.0321; 10.97; 0.723; 0.1046; 0.1249; 1.013. **8.** 0.706; 0.294; 0.412; 0.471; 0.823; 0.941. **9.** 11.6; 37.6; 20.6; 61.3; 9.25; 22.2. **10.** 0.428; 0.157; 0.0672; 0.1142; 0.01285; 0.1315; 0.900; 0.714.

Page 80

1. 2.13. **2.** 4.66. **3.** 4.72. **4.** 363. **5.** 1.77.

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1. 377. **2.** 0.262. **3.** 1.208. **4.** 0.288. **5.** 0.1955. **6.** 0.01296. **7.** 0.00218. **8.** 1.274. **9.** 5.02. **10.** 3.48.

Page 85

1. 2.52. **2.** 5.18. **3.** 0.0404. **4.** 3.71. **5.** 1.125. **6.** 94.3. **7.** 80.6. **8.** 0.90. **9.** 64.8. **10.** 14.7. **11.** 0.0578. **12.** 5.65. **13.** 5.66. **14.** 7,440. **15.** 12,920. **16.** 142.8. **17.** 2.93. **18.** 0.226. **19.** 130.6. **20.** 19.08. **21.** 0.375; 0.231; 0.1762. **22.** 0.555; 0.455; 0.417. **23.** 0.1905; 0.238; 0.524. **24.** 0.1263; 0.276; 0.356. **25.** 0.1285; 0.1745; 0.211. **26.** 9.73. **27.** 2.73. **28.** 0.203. **29.** 681. **30.** 7.29.

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1. 47. **2.** 0.313. **3.** 1.235. **4.** 0.651. **5.** 0.00232. **6.** 75.5. **7.** 0.234. **8.** 2.63. **9.** 31.3. **10.** 0.01193. **11.** 42,500. **12.** 1. **13.** 0.0242. **14.** 3.67. **15.** 12.94.

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1. 20.2. **2.** 6,590. **3.** 51.6. **4.** 0.0259. **5.** 4,540. **6.** 13,980. **7.** 23,700. **8.** 0.00108. **9.** 1.115. **10.** 6.38. **11.** 1.795. **12.** 3.87. **13.** 392. **14.** 124.7. **15.** 194.8. **16.** 0.00241. **17.** .1822. **18.** 10.24. **19.** 0.00383. **20.** 3. **21.** 0.685. **22.** 0.0963. **23.** 2.49. **24.** 0.53. **25.** 0.1159. **26.** 141. **27.** 0.536. **28.** 11.84. **29.** 0.818. **30.** 25.3.

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1. 305 sq. ft. **2.** 132.9 sq. in. **3.** 6.42 sq. cm. **4.** 5.04. **5.** 31.8 lb. **6.** 0.00541. **7.** 38,500. **8.** 1,030. **9.** 0.097. **10.** 27,800,000. **11.** 0.0000196. **12.** 75. **13.** 41,500. **14.** 0.00137. **15.** 400,000.

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1. right half. **2.** left half. **3.** center index. **4.** left half. **5.** left index. **6.** left half. **7.** center index. **8.** right half. **9.** right half. **10.** left index.

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1. 9.35 ft. **2.** 209 ft. **3.** 1.114 in. **4.** 0.259. **5.** 3.77 ft. **6.** 4,740. **7.** 1,497. **8.** 0.01868. **9.** 28.2. **10.** 0.442. **11.** 3.16. **12.** 0.0316. **13.** 761. **14.** 172. **15.** 0.00351.

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1. 529. **2.** 3.81. **3.** 38. **4.** 147,000. **5.** 0.77. **6.** 28,800,000. **7.** 1,740. **8.** 0.00515. **9.** 161. **10.** 3,200. **11.** 35.1. **12.** 9.68. **13.** 19.45. **14.** 49.2. **15.** 404. **16.** 0.286. **17.** 1.292. **18.** 0.857.

19. 4.63. **20.** 29.1. **21.** 41.2. **22.** 5.97. **23.** 104.1. **24.** 460. **25.** 11.1.

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1. 3,380. **2.** 19,700. **3.** 512. **4.** 343. **5.** 185,300. **6.** 6,850. **7.** 74,000. **8.** 729. **9.** 972,000. **10.** 1,157,000.

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1. 614. **2.** 1,859,000. **3.** 0.0001941. **4.** 86,800. **5.** 52.5. **6.** 0.0234. **7.** 1348. **8.** 0.802. **9.** 10.1. **10.** 0.0000000401. **11.** 3.79 cu. in. **12.** 1.239 cu. in. **13.** 4,780. **14.** 5,560 cu. ft. **15.** 13.35.

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1. ⅔. **2.** ⅓. **3.** 0. **4.** ⅔. **5.** 0. **6.** ⅓.

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1. 0. **2.** ⅓. **3.** 0. **4.** ⅔. **5.** ⅔. **6.** ⅓.

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1. 0.497. **2.** 5.72. **3.** 98.3. **4.** 8.07. **5.** 6.8. **6.** 0.909. **7.** 6.6. **8.** 9.52. **9.** 7.61. **10.** 8.6.

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1. 1.158. **2.** 1.76. **3.** 10.69. **4.** 1.908. **5.** 1.554. **6.** 2.10. **7.** 1.04. **8.** 16.52. **9.** 1.464. **10.** 2.03.

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1. 2.38. **2.** 3.75. **3.** 0.289. **4.** 0.456. **5.** 43.7. **6.** 4.07. **7.** 0.418. **8.** 3.68. **9.** 22.3. **10.** 4.46.

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1. 1.847 in. **2.** 17.9 ft. **3.** 8.18. **4.** 16.38. **5.** 0.579. **6.** 5.68. **7.** 1.895. **8.** 0.452. **9.** 50.9. **10.** 30.2. **11.** 0.0313. **12.** 0.1676. **13.** 8.12. **14.** 1.952. **15.** 22.3. **16.** 20.1. **17.** 877,000. **18.** 0.368. **19.** 177. **20.** 3,510. **21.** 0.000806. **22.** 2.05. **23.** 540. **24.** 27,800. **25.** 103.8. **26.** 3.35. **27.** 7.53. **28.** 0.881. **29.** 0.428. **30.** 17.4. **31.** 42.2. **32.** 1.82. **33.** 5.77. **34.** 10.65. **35.** 33.6.

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1. 930,000. **2.** 0.000650. **3.** 2,100,000. **4.** 41.1. **5.** 0.080. **6.** 5,200. **7.** 62,000. **8.** 0.770. **9.** 0.0084. **10.** 7,550. **11.** 379 cu. in. **12.** 0.00816 cu. in. **13.** 61.2 cu. in. **14.** 990. **15.** 106.

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1. 3.25. **2.** 7.01. **3.** 15.1. **4.** 0.1954. **5.** 31.8. **6.** 125. **7.** 17.4. **8.** 6.6. **9.** 0.755. **10.** 19.7. **11.** 4.73 in. **12.** 12.4 ft. **13.** 3.42. **14.** 9.14. **15.** 0.714 in.

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1. 60.5. **2.** 1,950. **3.** 0.357. **4.** 9,800. **5.** 0.150. **6.** 439. **7.** 104. **8.** 7,400. **9.** 5.45. **10.** 23,600. **11.** 3.27. **12.** 0.875. **13.** 5.33. **14.** 21.3. **15.** 1.398. **16.** 7.52. **17.** 4.46. **18.** 0.38. **19.** 1.212. **20.** 26.

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1. 0.920. **2.** 0.800. **3.** 0.1822. **4.** 0.866. **5.** 0.707. **6.** 0.0779. **7.** 0.447. **8.** 0.264. **9.** 0.0480. **10.** 0.956. **11.** 147.8 lb. **12.** 47 lb.

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1. 0.00940. **2.** 0.00741. **3.** 0.000348. **4.** 0.00320. **5.** 0.00577. **6.** 0.00408. **7.** 0.00240. **8.** 0.001875. **9.** 0.0001791. **10.** 0.00618. **11.** 10.1 lb. **12.** 0.0852 mi. or 450 ft. **13.** 11,440 ft. **14.** 0.206 in.

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1. 0.827. **2.** 0.903. **3.** 0.620. **4.** 0.676. **5.** 0.940. **6.** 0.0262. **7.** 0.00436. **8.** 0.513. **9.** 0.233. **10.** 0.412.

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1. 0.991. **2.** 0.950. **3.** 0.971. **4.** 0.990. **5.** 0.994. **6.** 0.958. **7.** 0.983. **8.** 0.980. **9.** 0.985. **10.** 0.968.

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1. −0.423. **2.** −0.766. **3.** 0.1765. **4.** 0.836. **5.** −0.887. **6.** −0.5. **7.** −0.152. **8.** 0.152. **9.** −0.625. **10.** −0.542.

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1. −0.866. **2.** −0.462. **3.** 0.195. **4.** 0.866. **5.** −0.171. **6.** −0.707. **7.** −0.737. **8.** −0.0987. **9.** 0.707. **10.** 0.924.

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1. 48°25′. **2.** 9°2′. **3.** 3°15′. **4.** 41°50′. **5.** 25°30′. **6.** 11°50′. **7.** 4°48′. **8.** 64°50′. **9.** 20°30′. **10.** 30°.

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1. 33′42″. **2.** 11′23″. **3.** 2′29″. **4.** 6′39″. **5.** 18′36″. **6.** 3′1″. **7.** 13′17″. **8.** 4′8″. **9.** 23′30″. **10.** 3′26″.

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1. 31°35′. **2.** 72°18′. **3.** 73°50′. **4.** 79°13′. **5.** 60°. **6.** 44°35′. **7.** 48°10′. **8.** 84°16′. **9.** 21°15′. **10.** 65°22′.

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1. 73°57′. **2.** 77°41′. **3.** 79°44′. **4.** 81°54′. **5.** 71°48′. **6.** 75°28′. **7.** 75°56′. **8.** 76°25′. **9.** 84°16′. **10.** 74°48′.

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1. 50°40′. **2.** 79°27′. **3.** 85°26′. **4.** 21°42′. **5.** 59°10′. **6.** 36°50′. **7.** 87°18′. **8.** 75°31′. **9.** 70°33′. **10.** 9°56′. **11.** 7°11′. **12.** 64°10′. **13.** 38°40′. **14.** 20°. **15.** 3°47′. **16.** 0.462. **17.** —0.584. **18.** —0.932. **19.** —0.151. **20.** 0.966. **21.** 0.999. **22.** —0.737. **23.** 0.0262. **24.** 0.639. **25.** —0.264. **26.** 0.339. **27.** —0.997. **28.** 0.375. **29.** —0.154. **30.** 0.0063.

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1. 0.667. **2.** 0.498. **3.** 0.313. **4.** 0.915. **5.** 0.1515. **6.** 0.1036. **7.** 0.368. **8.** 0.227. **9.** 0.766. **10.** 0.339. **11.** 57.8 ft. **12.** 5.55 in.

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1. 0.0960. **2.** 0.0323. **3.** 0.0588. **4.** 0.0698. **5.** 0.0819. **6.** 0.0421. **7.** 0.0218. **8.** 0.0888. **9.** 0.0686. **10.** 0.0381. **11.** 136 ft. **12.** 16.75 in.

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1. 0.00445. **2.** 0.01621. **3.** 0.00239. **4.** 0.00675. **5.** 0.01309. **6.** 0.0000872. **7.** 0.00361. **8.** 0.0112. **9.** 0.00504. **10.** 0.01731.

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1. 1.055. **2.** 3.13. **3.** 9.17. **4.** 1.192. **5.** 1.693. **6.** 4.61. **7.** 2.01. **8.** 1.00. **9.** 6.06. **10.** 10.00.

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1. 11.45. **2.** 14.13. **3.** 76.30. **4.** 10.90. **5.** 15.29. **6.** 21.70. **7.** 39.90. **8.** 68.80. **9.** 18.58. **10.** 20.45.

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1. 1.086. **2.** —1.102. **3.** —0.148. **4.** —0.245. **5.** —1.634. **6.** —0.698. **7.** 1.000. **8.** —5.77. **9.** 0.519. **10.** —1.000.

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1. 6°44′. **2.** 43°40′. **3.** 40°55′. **4.** 18°28′. **5.** 35°20′. **6.** 34°45′. **7.** 5°46′. **8.** 25°5′. **9.** 11°14′. **10.** 16°44′.

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1. 45°20′. **2.** 56°18′. **3.** 73°51′. **4.** 80°10′. **5.** 75°59′. **6.** 81°28′. **7.** 61°37′. **8.** 64°47′. **9.** 71°55′. **10.** 53°25′.

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1. 5°3′. **2.** 9′40″. **3.** 5°34′. **4.** 2°45′. **5.** 38′0″. **6.** 3′37″. **7.** 6′38″. **8.** 3°47′. **9.** 25′6″. **10.** 1°26′.

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1. 84°40′. **2.** 88°39′. **3.** 86°40′. **4.** 87°9′. **5.** 88°4′. **6.** 84°50′. **7.** 88°10′. **8.** 87°23′. **9.** 89°. **10.** 89°14′. **11.** 0.315. **12.** 2.21. **13.** 0.1028. **14.** 0.943. **15.** 4.39. **16.** 0.543. **17.** 0.0480. **18.** 6.53. **19.** 0.602. **20.** 14.96. **21.** 19°54′. **22.** 74°33′. **23.** 4°52′. **24.** 36°45′. **25.** 56°42′. **26.** 87°20′. **27.** 9°29′. **28.** 69°48′. **29.** 40°30′. **30.** 3°10′.

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1. 4.016. **2.** 1.719. **3.** 8.850—10. **4.** 0.386. **5.** 9.783—10. **6.** 2.131. **7.** 0.295. **8.** 7.471. **9.** 4.813—10. **10.** 2.941.

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1. 673. **2.** 3.51. **3.** 16.30. **4.** 0.222. **5.** 1,019. **6.** 0.01567. **7.** 20,300.0. **8.** 0.891. **9.** 11.43. **10.** 3.05.

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1. 5.313. **2.** 1.69. **3.** 3.62. **4.** —0.483. **5.** 2.30. **6.** —2.87. **7.** 4.92. **8.** 9.90. **9.** 0.583. **10.** 6.77.

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1. 65.4. **2.** 2.04. **3.** 3.57. 4. 0.0369. **5.** 10.00. **6.** 0.100. **7.** 19.13. 8. 5.98. **9.** 0.514. **10.** 41.3.

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1. 10.58. **2.** 773. **3.** 41.0. 4. 150.9. **5.** 10,310. **6.** 0.361. **7.** 750. 8. 112.6. **9.** 32.5. **10.** 0.0792. **11.** 276,000. **12.** 95,700. **13.** 81,800. **14.** 143.1. **15.** 19,520.

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1. 233. **2.** 11.60. **3.** 0.141. 4. 210. **5.** 0.0631. **6.** 0.140. **7.** 18.72. 8. 0.01093. **9.** 13.87. **10.** 3.03. **11.** 1.208. **12.** 0.000575. **13.** 0.00439. **14.** 2.58. **15.** 2.18.

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1. 4,360. **2.** 39.5. **3.** 239. 4. 19,300. **5.** 670,000. **6.** 40.8. **7.** 2.63. **8.** 151.5. 9. 61.5. **10.** 0.523. **11.** 273. **12.** 190.5. **13.** 7,240. **14.** 12. **15.** 0.548.

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1. 0.376. **2.** 2.46. **3.** 1.127. 4. 3.52. **5.** 35.9. **6.** 0.348. **7.** 2,420. 8. 0.336. **9.** 8.51. **10.** 10.6. **11.** 3,980. **12.** 0.0001028. **13.** 0.77. **14.** 2.54. **15.** 152.6.

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1. 941. **2.** 6.47. **3.** 32,500. 4. 89,800. **5.** 57. **6.** 592. **7.** 0.253. 8. 19.7. 9. 169.7. **10.** 3.23. **11.** 1.777. **12.** 183. **13.** 159.2. **14.** 9.53. **15.** 1.4. **16.** 0.917. **17.** 0.323. **18.** 12.52. **19.** 0.00762. **20.** 69.8.

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MATHEMATICS FOR INDUSTRY

By

S. E. Rusinoff

This book was developed to give you the mathematics you need in industry. Every principle introduced has an ultimate application to a practical shop problem—problems that experience has shown are most frequently encountered.

A conspicuous quality of this book is its simplicity and clarity. Each principle is developed in a step-by-step sequence. Each problem is selected for its practicality.

One of the many unusual features is the chapter on Engineering Computations. This chapter presents simple methods of making many of those computations which otherwise would have to be referred to engineers.

Machinists, precision inspectors, tool-and-die makers, draftsmen, tool designers, time-study men, industrial engineers, and many others will find this book their most valuable "tool" and their most practical source of time-saving information.